DFG-Sonderforschungsbereich 393

"Numerische Simulation auf massiv parallelen Rechnern"

and

Fakultät für Mathematik, Technische Universität Chemnitz

Chemnitz FEM-Symposium 2002

Programme

Collection of abstracts

and

List of participants

Chemnitz, September 23 - 25, 2002

Programme for Monday, September 23, 2002

Start at 09:00

Chairman: Arnd Meyer

09:00 – 09:05	A. Meyer (Chemnitz) Welcome
09:05 – 09:50	J. Schöberl (Linz) Anisotropic Mesh Generation and Mesh Refinement with Netgen
10:00 – 10:25	V. Dolejsi (Prague) An alternative formulation of the anisotropic mesh adaptation method

---

Tea and coffee break

Chairman: Gert Lube

10:45 – 11:35	L. Formaggia (Milano) Some anisotropic mesh adaption strategies for FEM
11:45 – 12:10	S. Micheletti (Milano) Some remarks on the stability coefficients on anisotropic meshes with application to the advection-diffusion and Stokes problems
12:15 – 12:40	G. Kunert (Chemnitz) A posteriori error estimation for the Stokes problem on anisotropic FE discretizations

---

Lunch

Chairman: Dietrich Braess

14:30 – 14:55	T. Apel (Chemnitz) Stable finite elements for the Stokes problem on anisotropic meshes
15:00 – 15:25	G. Matthies (Magdeburg) The inf-sup condition for the family of mapped Q_k/P_{k-1}^{disc} finite element pairs, $k \geq 2$
15:30 – 15:55	G. Lube (Göttingen) Finite element calculation of incompressible flows using div-stable elements and SUPG-stabilization
16:00 – 16:25	T. Richter (Heidelberg) Parallel and adaptive finite elements for the Navier-Stokes equation

---

Tea and coffee break

Chairman: Bernd Heinrich

16:45 – 17:10	V. Heuveline (Heidelberg) On the numerical simulation of the free fall problem
17:15 – 17:55	M. Bause, F. Radu (Erlangen) Finite Element Approximation of Variably Saturated Subsurface Flow with Reactive Transport
18:00 – 18:25	M. Braack, (Heidelberg) A posteriori control of modeling errors and discretization errors
18:30 – 19:00	O. Steinbach (Stuttgart) A robust boundary element method for nearly incompressible linear elasticity

---

Dinner

20:00 Wine reception

Programme for Tuesday, September 24, 2002

Start at 09:00

Chairman: Joachim Schöberl

09:00	–	09:50	H. Blum (Dortmund) A general concept for a posteriori error control for variational inequalities
10:00	–	10:25	F. T. Suttmeier (Dortmund) Error bounds for Finite Element Solutions of Elliptic Variational Inequalities of second kind
10:30	–	10:55	R. Unger (Chemnitz) Projection methods for contact problems in elasticity

---

Tea and coffee break

Chairman: Luca Formaggia

11:15	–	11:40	D. Braess (Bochum) Cascadic Aspects of Multigrid Methods for Unilateral Problems
11:45	–	12:10	B. Heinrich (Chemnitz) Nitsche-type mortaring for a singularly pertubed reaction-diffusion problem
12:15	–	12:40	B. Flemisch (Stuttgart) Coupling scalar and vector potentials on non-matching grids for eddy currents modelling in moving conductors
12:45	–	13:10	B. Lamichhane (Stuttgart) Higher Order Dual Lagrange Multiplier Spaces for Mortar Finite Element Discretizations

Lunch

14:45			Excursion to silvermine ”Sauberg”

Dinner

Chairman: Wolfgang Dahmen

20:00	–	20:25	F. Hülsemann (Erlangen) Multigrid methods on hierarchical hybrid grids
20:30	–	20:55	J. Kienesberger (Linz) A Multigrid Preconditioned Solver for Elastoplastic Type Problems
21:00	–	21:25	C. Pester (Chemnitz) Efficient methods to solve a quadratic operator eigenvalue problem
21:30	–	21:55	M. Maischak (Chemnitz) Mixed FEM and BEM Coupling for a Linear Transmission Problem with a Signorini Interface

Programme for Wednesday, September 25, 2002

Start at 09:00

Chairman: Heribert Blum

09:00	–	09:50	W. Dahmen (Aachen) Adaptive Techniques – Complexity and Error Estimates
10:00	–	10:25	H. Harbrecht (Chemnitz) Wavelet Based Fast Solution of BEM
10:30	–	10:55	R. Stevenson (Utrecht) Adaptive solution of operator equations using wavelet frames

Tea and coffee break

Chairman: Stefano Micheletti

11:15	–	11:40	S. Beuchler (Chemnitz) Multi-resolution weighted norm equivalences and applications
11:45	–	12:10	G. Winkler (Chemnitz) Calculation of Call Prices under Stochastic Volatility
12:15	–	12:40	A. Rababah (Irbid) High accuracy piecewise approximation for planar curves
12:45	–	13:10	B. Vexler, (Heidelberg) Adaptive Finite Element Methods for Parameter Identification Problems
13:15	–	13:20	A. Meyer (Chemnitz) Closing

Lunch

Stable finite elements for the Stokes problem on anisotropic meshes

Thomas Apel[1]

Anisotropic meshes are characterized by elements with a large or even asymptotically unbounded aspect ratio. Such meshes are known to be particularly effective for the resolution of directional features of the solution, for example edge singularities and boundary layers.

In solving the Stokes or Navier-Stokes problem the question arises which pairs of elements are stable independent of the aspect ratio of the elements. For several pairs of elements positive and negative results are given.

References:

[1] Th. Apel and S. Nicaise and J. Schöberl: A non-conforming finite element method with anisotropic mesh grading for the Stokes problem in domains with edges. IMA J. Numer. Anal. 21 (2001), 843–856

[2] Th. Apel and M. Randrianarivony: Stability of discretizations of the Stokes problem on anisotropic meshes. To appear in Mathematics and Computers in Simulation.

[1]TU Chemnitz, Fak. f. Mathematik, 09107 Chemnitz, Germany, na.apel@na-net.ornl.gov

Finite Element Approximation of Variably Saturated Subsurface Flow with Reactive Transport

Markus Bause[1] Florin Adrian Radu[2]

Accurate, reliable and efficient simulations of moisture fluxes through porous media are desirable in hydrological and environmental studies, as well as in civil and environmental engineering. The nonlinear degenerate Richards' equation as a model for saturated-unsaturated flow in porous media is the basic equation of simulation tools for flow and transport processes in the subsurface. Here, an adaptive mixed hybrid finite element discretization of the Richards' equation is presented. Two different kinds of error indicators for space adaptive grid refinement are considered which can be calculated easily by means of the available finite element approximations. The effectiveness and robustness of the approach is illustrated by computational experiments conducted for realistic water table recharge problems. Then, the Richards' equation is coupled with some model for reactive solute transport. The finite element discretization techniques used for the set of coupled convection-diffusion-reaction equations and the algorithmic solution process of the resulting nonlinear algebraic equations are presented. Finally, illustrative numerical studies are presented for realistic problems.

[1]Universität Erlangen-Nürnberg, Institut für Angewandte Mathematik, Martensstr. 3, 91058 Erlangen, Deutschland,
bause@am.uni-erlangen.de

[2]Universität Erlangen-Nürnberg, Institut für Angewandte Mathematik, Martensstr. 3, 91058 Erlangen, Deutschland,
raduf@am.uni-erlangen.de

Multi-resolution weighted norm equivalences and applications

Sven Beuchler[1] Reinhold Schneider[2] Christoph Schwab[3]

We establish norm equivalences in weighted spaces $L^2_w((0,1))$ with possibly singular weight functions $w(x) \geq 0$ in $(0,1)$. Our analysis exploits the locality of the biorthogonal wavelet basis and its dual basis functions. The discrete norms are sums of wavelet coefficients which are weighted with respect to the collocated weight function $w(x)$ within each scale. Since norm equivalences for Sobolev norms are by now well-known, our result can also be applied to weighted Sobolev norms. We apply our theory to the problem of preconditioning p-Version FEM.

[1]TU Chemnitz, Fakultät für Mathematik, 09107 Chemnitz, Germany,
sven.beuchler@mathematik.tu-chemnitz.de

[2]TU Chemnitz, Fakultät für Mathematik, 09107 Chemnitz, Germany,
reinhold@mathematik.tu-chemnitz.de

[3]ETHZ, Seminar of Applied Mathematics, 8092 Zürich, Switzerland,
christoph.schwab@sam.math.ethz.ch

A general concept for a posteriori error control for variational inequalities

Heribert Blum[1]

The concept of weighted a posteriori error bounds has originally been developed for nonlinear variational equations and successfully been employed in many fields of applications (Rannacher et.al.). The main advantage of this duality-based approach is its flexibility with respect to the desired measure for the error which can be described by an arbitrary functional and thus can easily be adapted to the needs of the user.

Based on joint work with F.T.Suttmeier we here present a generalization to classes of problems leading to variational inequalities, arising, e.g., from problems in elastomechanics with one-sided side conditions. The key ingredient is a suitable adaptation of the duality argument to this situation which is based on an idea of Natterer. The efficiency and reliability of our approach is illustrated by several numerical tests for problems of obstacle and Signorini type stemming from applications in mechanical engineering.

[1]Universität Dortmund, Fachbereich Mathematik , D-44221 Dortmund,
heribert.blum@math.uni-dortmund.de

A posteriori control of modeling errors and discretization errors

Malte Braack[1] Alexandre Ern[2]

We investigate the concept of dual-weighted residuals for measuring model errors in the numerical solution of nonlinear partial differentialal equations. The method is derived to handle simultaneously model and discretization errors. We present an adaptive model/mesh refinement procedure where both sources of errors are equilibrated. Various test cases involving Poisson equations and convection-diffusion-reaction equations with complex diffusion models (highly oscillatory diffusion coefficient, nonlinear turbulent viscosity, multicomponent diffusion matrix) confirm the reliability of the analysis and the efficiency of the proposed methodology.

[1]Universität Heidelberg, Institut für Angewandte Mathematik, 69120 Heidelberg,
malte.braack@iwr.uni-heidelberg.de

[2]CERMICS, Ecole nationale des ponts et chaussees, 6 et 8, av. Blaise Pascal, 77455 Marne la Vallee cedex 2, France,
ern@cermics.enpc.fr

Cascadic Aspects of Multigrid Methods for Unilateral Problems

Dietrich Braess[1] Heribert Blum[2] Franz-Theo Suttmeier[3]

When an obstacle problem or the Signorini problem is to be solved. we have a variational problem in a convex cone (and not in a linear space). Therefore, one cannot proceed as in classical multigrid procedures. Either an inner or an outer approximation of the cones is used on the coarser levels. An inspection of existing codes shows that this drawback is often circumvented by using tools from cascadic procedures. We design the algorithm so that it becomes consistent with cascadic multigrid methods and those cases for which a complete theory exists.

[1]Ruhr-University, Faculty of Mathematics, D 44780 Bochum,
braess@num.ruhr-uni-bochum.deRuhr-Universit

[2]University of Dortmund,
Heribert.Blum@Math.Uni-Dortmund.de

[3]University of Dortmund,
Franz-Theo.Suttmeier@math.uni-dortmund.de

Adaptive Techniques – Complexity and Error Estimates

Wolfgang Dahmen[1]

Adaptive discretizations form a key methodology for treating large scale numerical simulation problems derived from partial differential or singular integral equations. This talk highlights some recent developments concerning complexity and convergence estimates for adaptive solvers mainly obtained in collaboration with A. Cohen, R. DeVore and also P. Binev. The central question can be phrased as follows. Suppose that the solution can be approximated (within the given discretization framework) with accuracy ϵ at best (with full information) at the expense of $N(\epsilon)$ degrees of freedom, can one devise a concrete adaptive scheme that produces, for every target accuracy ϵ, an approximate solution at the expense of the same asymptotic work rate $N(\epsilon)$, uniformly in $\epsilon > 0$?

Only recently such results (the first of this type at all) could be established for a new class of wavelet based schemes and for a wide class of variational problems including indefinite and also nonlinear problems. An analogous statement can be shown to hold in the finite element context for second order elliptic boundary value problems on planar domains. In this talk some of these developments and the respective main conceptual ingredients are outlined.

References:

[1] A. Cohen, W. Dahmen, R. DeVore, Adaptive wavelet methods for elliptic operator equations – Convergence rates, Math. Comp. 70 (2001), 27–75.
[2] A. Cohen, W. Dahmen, R. DeVore, Adaptive wavelet methods II - Beyond the elliptic case, IGPM Report # 199, RWTH Aachen, Nov. 2000, to appear in Foundations of Computational Mathematics.
[3] S. Dahlke, W. Dahmen, K. Urban, Adaptive wavelet methods for saddle point problems – Convergence rates, IGPM Report # 204, RWTH Aachen, July 2001, to appear in SIAM J. Numer. Anal..
[4] A. Cohen, W. Dahmen, R. DeVore, Adaptive Wavelet Schemes for Nonlinear Variational Problems, IGPM Report, RWTH Aachen, July 2002.
[5] P. Binev, W. Dahmen, R. DeVore, Adaptive Finite Element Methods with Convergence Rates, IGPM Report, RWTH Aachen, June 2002.

[1]IGPM, RWTH Aachen, 52056 Aachen, Germany,
dahmen@igpm.rwth-aachen.de

An alternative formulation of the anisotropic mesh adaptation method.

Vit Dolejsi[1]

During last years we dealt with the use of the anisotropic mesh adaptation (AMA) method for the numerical simulation of compressible flows. We derived a necessary condition for a grid and we showed how its satisfaction can be used for a mesh adaptation algorithm. The essential terms were *optimal triangle* and its (rather heurestic) extension *optimal mesh*.

Our presented formulation allows us define the *optimal mesh* without the term *optimal triangle* and then we avoid that heurestic step. Therefore the new formulation is simpler and mathematically more rigorous. Moreover this approach can be easily applied for numerical analysis.

For a given fucntion u and a mesh τ_h we define the piecewise linear projection $r_h u$ such that

$$r_h u(x_i) = u(x_i), \qquad \nabla r_h u(x_i) = \nabla u(x_i) \qquad \forall T_i \in \tau_h,$$

where x_i is barycentre of T_i. Then our aim is to control the *interpolation error function*

$$E_I(x) \equiv |u(x) - r_h u(x)| \leq \omega,$$

whera $\omega > 0$ is a given tolerance. Considering E_i over edges of τ_h we derive an *edge optimization criterion* which is used for mesh adaptation.

The anisotropic mesh adaptation technique and its implementation will be described. Several (2D as well as 3D) numerical examples will be presented.

[1]Charles University Prague, Faculty of Mathematics and Physics, Sokolovska 83, 186 75 Prague, Czech Republic,
dolejsi@karlin.mff.cuni.cz

Coupling scalar and vector potentials on non-matching grids for eddy currents modelling in moving conductors

Bernd Flemisch[1] Yvon Maday[2] Francesca Rapetti[3] Barbara I. Wohlmuth[4]

The $T - \Omega$ formulation of the magnetic field is a widely used approach for the approximation of the magnetic quantities modelled by the eddy current equations. This decomposition allows to use a scalar function in the main part of the computational domain, reducing the use of vector quantities to the conducting parts. We propose here to approximate these two quantities on different and non-matching grids to be able for instance to tackle a problem where the conducting part can move inside the global domain. The connection between the two grids is managed with the mortar element tools. The talk will focus on the implementation of the resulting algorithm and on the presentation of numerical results.

References:

[1] to be published in a special volume of Journal CAM

[1]Universität Stuttgart, Mathematisches Institut A, , Pfaffenwaldring 57, 70569 Stuttgart, Germany, flemisch@mathematik.uni-stuttgart.de

[2]Laboratoire J.-L. Lions, CNRS & Université Pierre-et-Marie-Curie, , Boîte 187, 4, place Jussieu, 75252 Paris cedex 05, France,
maday@ann.jussieu.fr

[3]Laboratoire J.-A. Dieudonné, CNRS & Université de Nice et , Sophia-Antipolis, Parc Valrose, 06108 Nice cedex 02, France,
frapetti@math.unice.fr

[4]Universität Stuttgart, Mathematisches Institut A, Pfaffenwaldring 57, 70569 Stuttgart, Germany
wohlmuth@mathematik.uni-stuttgart.de

Some anisotropic mesh adaption starategies for FEM

Luca Formaggia[1]

We present a general framework for anisotropic mesh adaption for linear finite elements.

We will present some a-priori results for the interpolation error and discretisation error for eliptic problems on anisotropic grids. A posteriori results are obtained by controlling a functional of the solution, extending the strategy by Rannacher and coworkers to anisotropic meshes.

The technique will be applied to elliptic, convection-diffusion and Stokes problems.

[1]SB/IMA EPFL Lausanne and, Politecnico di Milano, Italy,
luca.formaggia@epfl.ch

Wavelet Based Fast Solution of BEM

Helmut Harbrecht[1] Reinhold Schneider[2]

Solving a boundary integral equation by the Galerkin scheme leads to a densely populated system matrix which is often ill conditioned. Thus, the computation of the solution requires at least $\mathcal{O}(N_J^2)$ operations, where N_J denotes the number of unknowns. This makes the boundary element method unattractive for the practical usage.

In the last years fast algorithms, like the Fast Multipole Method and the Panel Clustering, have been developed to reduce the complextity considerably. Another fast method is the wavelet Galerkin scheme: one employs biorthogonal wavelet bases with vanishing moments for the discretization of the given boundary integral equation. The resulting system matrix is quasi sparse and can be compressed without loss of accuracy to only $\mathcal{O}(N_J)$ nonzero entries.

The present talk is concerned with the principles as well as new developments of the wavelet Galerkin scheme for boundary integral equations.

[1]Fakultät für Mathematik, Technische Universität Chemnitz, D-09107 Chemnitz, helmut.harbrecht@mathematik.tu-chemnitz.de

[2]Fakultät für Mathematik, Technische Universität Chemnitz, D-09107 Chemnitz, reinhold.schneider@mathematik.tu-chemnitz.de

Nitsche-type mortaring for a singularly pertubed reaction-diffusion problem

Bernd Heinrich[1] Kornelia Pietsch[2]

The paper is concerned with Nitsche-type mortaring for treating weak continuity across non-matching meshes for domain decomposition, in particular for the numerical treatment of a singularly pertubed reaction-diffusion problem in polygonal domains. The interface of the non-matching meshes is related with the boundary layers resolved by anisotropic meshes. Some properties of the finite element schemes as well as the convergence are proved and illustrated by numerical experiments.

[1]TU Chemnitz, Fakultät für Mathematik , PSF 964, 09107 Chemnitz, Germany, heinrich@mathematik.tu-chemnitz.de

[2]TU Chemnitz, Fakultät für Mathematik, 09107 Chemnitz, Germany, kornelia.pietsch@mathematik.tu-chemnitz.de

A posteriori error control and adaptive mesh design for the free steady fall problem

Vincent Heuveline[1]

The numerical simulation of the free fall of a solid body in a viscous fluid is a challenging task since it requires computational domains which usually need to be several order of magnitude larger than the solid body in order to avoid the influence of artificial boundaries. Toward an optimal mesh design in that context, we propose a method based on the *weighted* a posteriori error estimation of the finite element approximation of the fluid/body motion. A key ingredient for the proposed approach is the reformulation of the conservation and kinetic equations in the solid frame as well as the implicit treatment of the hydrodynamic forces and torque acting on the solid body in the weak formulation. Informations given by the solution of an adequate dual problem allow to control the discretization error of given functionals. The analysis encompasses the control of the free fall velocity, the orientation of the body, the hydrodynamic force and torque on the body. Numerical experiments for the two dimensional sedimentation problem validate the method.

[1]University Heidelberg, Numerical Analysis Group,
vincent.heuveline@iwr.uni-heidelberg.de

Multigrid methods on hierarchical hybrid grids

Frank Hülsemann[1] Ben Bergen[2] Ulrich Rüde[3]

The hierarchical hybrid grid data structure consists of a hierarchy of block-structured refinements of a given, potentially unstructured input grid. This grid structure offers the geometric flexibility of unstructured grids combined with the possibility to exploit the patchwise regularity for efficient implementations. The design targets for this approach were multigrid methods on high performance architectures. The talk presents the key aspects of the grid framework that are relevant for high performance computing and applications of multigrid schemes on such grids.

[1]System Simulation Group, Computer Science Department, University Erlangen, Cauerstrasse 6, 90158 Erlangen, Germany,
frank.huelsemann@cs.fau.de

[2]System Simulation Group, Computer Science Department, , University Erlangen,
Ben.Bergen@cs.fau.de

[3]System Simulation Group, Computer Science Department, , University Erlangen,
Ulrich.Ruede@cs.fau.de

A Multigrid Preconditioned Solver for Elastoplastic Type Problems

Johanna Kienesberger[1]

In this talk we discuss an efficient solution method for problems of elastoplasticity.

The phenomenon of plasticity is modeled by an additional term in the stress-strain relation, the evolution of this additional term in time is described by the Prandtl-Reuss normality law. After discretizing the problem in time, we derive a dual formulation. Our solution algorithm is based on an equivalent minimization problem, which is presented for an isotropic hardening law.

Since the objective is not differentiable, we use a differentiable, piecewise quadratic regularization.

The algorithm is a sucessive sub-space optimization method: In the first step, we solve a Schur-complement system for the displacement variable by a multigrid preconditioned conjugate gradient method. The second step, namely the minimization in the plastic part of the strain, splits into a large number of local optimization problems.

The numerical tests show the fast convergence of the presented algorithm.

[1]University of Linz, SFB 013, Freistaedterstrasse 313, 4040 Linz, Austria,
johanna.kienesberger@jku.at

A posteriori error estimation for the Stokes problem on anisotropic FE discretizations

Gerd Kunert[1] Serge Nicaise[2] Emmanuel Creusé[3]

The Stokes problem can yield solutions with singularities of *anisotropic* behaviour, e.g. along concave edges in three spatial dimensions. Then solution adapted, anisotropic discretizations can be advantageous. However, known isotropic *a posteriori* error estimators have to be modified to provide reliable results on anisotropic meshes.

We present recent proposals for such anisotropic error estimators. The research covers both 2D and 3D domains as well as conforming and non-conforming discretizations. The choice of the discrete spaces turns out to be particularly important.

Some numerical results accompany the theory.

[1]TU Chemnitz, Fakultät für Mathematik, 09107 Chemnitz, Germany,
gerd.kunert@mathematik.tu-chemnitz.de

[2]Université de Valenciennes et du Hainaut Cambrésis, MACS, B.P. 311 , 59304 Valenciennes Cedex, France,
serge.nicaise@univ-valenciennes.fr

[3]Université de Valenciennes et du Hainaut Cambrésis, MACS, B.P. 311 , 59304 Valenciennes Cedex, France,
Emmanuel.Creuse@univ-valenciennes.fr

Higher Order Dual Lagrange Multiplier Spaces for Mortar Finite Element Discretizations

Bishnu Prasad Lamichhane[1] Barbara Wohlmuth[2]

Domain decomposition techniques provide a powerful tool for the numerical approximation of partial differential equations. Nonconforming domain decomposition techniques provide a more flexible approach than standard conforming formulations. They are of special interest for time-dependent problems, diffusion coefficients with jumps, problems with local anisotropies, corner singularities, and when different terms dominate in different regions of the simulation domain. To obtain a stable and optimal discretization scheme for the global problem, the information transfer and the communication between the subdomains are of crucial importance. In a first part, the talk will concentrate on mortar techniques for quadratic finite elements, in particular, with dual Lagrange multiplier spaces in 2D. We will present numerical results for linear and quadratic mortar finite elements in 2D. Different Lagrange multiplier spaces will be compared. In the second part of the talk, we will discuss dual Lagrange multiplier spaces for quadratic finite elements in 3D both in hexahedral and tetrahedral cases. Finally, numerical results for linear and quadratic mortar elements in 3D will be presented.

References:

[1] Higher Order Dual Lagrange Multiplier Spaces for Mortar Finite Element Discretizations,Report Math. Inst. 2002-11

[1]Universität Stuttgart, Mathematical Institute A, Lehrstuhl 7,, Pfaffenwaldring 57, 70569 Stuttgart, lamichhane@mathematik.uni-stuttgart.de

[2]Universität Stuttgart, Mathematical Institute A, Lehrstuhl 7,, Pfaffenwaldring 57, 70569 Stuttgart, wohlmuth@mathematik.uni-stuttgart.de

Finite element calculation of incompressible flows using div-stable elements and SUPG-stabilization

Gert Lube[1]

Conforming finite element schemes are considered for the incompressible Navier-Stokes equations with the nonlinear terms written in the convection form. Implicit time integration together with streamline upwind (SUPG) stabilization in the case of high Reynolds numbers result in nice stability properties of linearized problems which can be solved by efficient numerical algorithms. The paper presents the stability and convergence analysis, including the design of stabilization parameters, for linearized equations using div-stable velocity-pressure approximation. Surprisingly, such results are seemingly not available in the literature. Numerical experiments confirm the theoretical results.

[1]University of Göttingen, Faculty of Mathematics, Lotzestrasse 16-18, D-37083 Göttingen, Germany, lube@math.uni-goettingen.de

Mixed FEM and BEM Coupling for a Linear Transmission Problem with a Signorini Interface

Matthias Maischak[1]

We are concerned with the mixed formulation of the interface problem consisting of a linear partial differential equation in $\Omega \subset \mathbb{R}^n$ (bounded, Lipschitz, $n \geq 2$) and the Laplace equation with some radiation condition in the unbounded exterior domain $\Omega_c := \mathbb{R}^n \backslash \bar{\Omega}$. The equations are coupled by transmission conditions and Signorini contact conditions on the interface $\partial\Omega$. The exterior part of the interface problem is rewritten using a Neumann to Dirichlet mapping (NtD) given in terms of boundary integral operators. We treat the general numerical approximation of the resulting variational inequality and discuss the non-trivial discretization of the NtD mapping. Assuming some abstract approximation properties and a discrete inf-sup condition we proof existence and uniqueness and show an a-priori estimate. Choosing Raviart-Thomas elements and piecewise constants in Ω and hat functions on $\partial\Omega$, we can apply a discrete inf-sup condition. We suggest a solver based on a modified Uzawa algorithm and show convergence. Finally we present some numerical results underlining our theory and compare with the primal formulation.

[1]TU Chemnitz, Fakultät für Mathematik, Reichenhainer Str.41, 09107 Chemnitz, Germany, maischak@ifam.uni-hannover.de

The inf-sup condition for the family of mapped Q_k/P_{k-1}^{disc} finite element pairs, $k \geq 2$

Gunar Matthies[1] Lutz Tobiska[2]

Pairs of finite element spaces which are used for calculations of incompressible flows should fulfil the inf-sup condition. Often discontinuous pressure approximations are preferred since they fulfil the incompressibility constraint on the discrete level more locally. Our talk will focus on finite elements on quadrilaterals and hexahedra.

First we recall the inf-sub-stability of the Bernardi–Raugel element [1] on simplices and quadrilaterals. Then, we extend it to hypercubes in any space dimension and show the inf-sup stability for the case where the hypercubes are bounded by hyperplanes only. Moreover, in the three-dimensional case, the inf-sup condition will be proven even for hexahedra with curved faces, see [5]. These hexahedra are the image of the unit cube under a general multilinear and bijective mapping.

For the Q_k/P_{k-1}^{disc} pairs of finite elements, $k \geq 2$, two possible versions exist. One can either use an unmapped version of P_{k-1}^{disc} which consits of piecewise polynomial functions of degree less than or equal to $k-1$ on each cell or one uses a mapped version where the local function space is obtained by transforming a polynomial space from the reference cell to the considered cell. Since the reference transformation is in general not affine but multilinear the two variants differ. It is well-known that the inf-sup condition is satisfied for the mapped-Q_k/unmapped-P_{k-1}^{disc} pairs, $k \geq 2$, see [4]. Recently, it has been shown by an algebraic argument in [3] that also the mapped-Q_2/mapped-P_1^{disc} pair on quadrilaterals fulfils the inf-sup condition. We will show that by applying the macro-technique of Boland and Nicolaides [2] and by using the extended Bernardi–Raugel element that the mapped-Q_k/mapped-P_{k-1}^{disc} pairs satisfies the inf-sup condition for arbitrary $k \geq 2$ in any space dimension, see [5].

References:

[1] C. Bernardi and G. Raugel, *Analysis of some finite elements for the Stokes problem*, Math. Comp., 44 (1985), pp. 71–79.
[2] J. Boland and R. Nicolaides, *Stability of finite elements under divergence constraints*, SIAM J. Numer. Anal., 20 (1983), pp. 722–731.
[3] D. Boffi and L. Gastaldi, *On the quadrilateral $Q_2 - P_1$ element for the Stokes problem*, Preprint 1194/00, Istituto di Analisi Numerica - C.N.R., Pavia, 2000.
[4] V. Girault and P.-A. Raviart, *Finite Element Methods for Navier–Stokes equations*, Springer-Verlag, Berlin-Heidelberg-New York, 1986.
[5] G. Matthies and L. Tobiska, *The inf-sup condition for the mapped Q_k/P_{k-1}^{disc} element in arbitrary space dimensions*, to appear in Computing, DOI 10.0107/s00607-002-1451-3.

[1]Otto-von-Guericke-Universität Magdeburg, Institut für Analysis und Numerik, PSF 4120, 39016 Magdeburg, Germany,
matthies@mathematik.uni-magdeburg.de

[2]Otto-von-Guericke-Universität Magdeburg, Institut für Analysis und Numerik, PSF 4120, 39016 Magdeburg, Germany,
tobiska@mathematik.uni-magdeburg.de

Some remarks on the stability coefficients on anisotropic meshes with application to the advection-diffusion and Stokes problems

Stefano Micheletti[1] Simona Perotto[2]

This work is motivated by CFD applications exhibiting directional features, e.g. boundary and internal layers or shocks in the case of the Navier-Stokes or Euler equations. In this kind of applications the use of stabilized finite elements and of suitably adapted meshes is mandatory.

In this regard, we have developed a theoretical anisotropic framework which, moving from the spectral properties of the affine mapping from the reference to the generic triangle, provides us with anisotropic interpolation error estimates for piecewise linear finite elements [2].

In this communication special emphasis will be cast over the design of the stability coefficients in the non-trivial case of (possibly) highly stretched elements. In particular, starting from the anisotropic error estimates developed in [2,3], we derive a priori error estimates with respect to some mesh dependent norms in the case of the advection-diffusion and the Stokes problems, for which we devise a new recipe for the stability coefficients [3]. These results have been recently improved in [4] where a comparison with the bubble stabilization approach is carried out (e.g. [1]). Our theoretical analysis is corroborated by several numerical examples.

References:

[1] F. Brezzi, L.P. Franca, T.J.R. Hughes and A. Russo. $b = \int g$. Comput. Methods Appl. Mech. Engrg. 145, 329-339 (1997).
[2] L. Formaggia and S. Perotto. New anisotropic a priori error estimates. Numer. Math. 89, 641-667 (2001).
[3] S. Micheletti, S. Perotto and M. Picasso. Stabilized finite elements on anisotropic meshes: a priori error estimates for the advection-diffusion and Stokes problems. SIAM J. Numer. Anal. (2002). Submitted.
[4] S. Micheletti, S. Perotto and M. Picasso. Some remarks on the stability coefficients and bubble stabilization of FEM on anisotropic meshes. Comput. Vis. Sci. (2002). Submitted.

[1]MOX - Modeling and Scientific Computing, Politecnico di Milano, Dipartimento di Matematica "F. Brioschi", via Bonardi 9, I-20133 Milano, Italy,
mike@mate.polimi.it

[2]MOX - Modeling and Scientific Computing, Politecnico di Milano, Dipartimento di Matematica "F. Brioschi", via Bonardi 9, I-20133 Milano, Italy,
simona@mate.polimi.it

Efficient methods to solve a quadratic operator eigenvalue problem

Cornelia Pester[1] Thomas Apel[2] David Watkins[3]

The stress distribution at the top of a polyhedral corner or at a crack tip has the typical r^α-singularity. Mathematically, the exponent α is an eigenvalue of a quadratic operator eigenvalue problem. The finite element method is sufficiently flexible to solve the problem numerically, such that also anisotropic or composite materials can be treated.

In this talk we introduce different methods to solve the corresponding matrix eigenvalue problem and compare their efficiency.

[1]TU Chemnitz, Fakultät für Mathematik, 09107 Chemnitz,
cornelia.pester@mathematik.tu-chemnitz.de

[2]TU Chemnitz, Fakultät für Mathematik, 09107 Chemnitz,
thomas.apel@mathematik.tu-chemnitz.de

[3]Washington State University, Department of Mathematics, Pullman, WA , 99164-3113 USA,
watkins@math.wsu.edu

High accuracy piecewise approximation for planar curves

Abedallah Rababah[1]

A piecewise approximation method is described for planar curves. The order of classical piecewise approximations is improved. The method exploits the freedom in the choice of the parametrization and rises the approximation order up to $\frac{4m}{3}$ where $m \geq 6$ is the degree of the approximating polynomial parametrization. For the case of $m = 5$, a quintic polynomial curve is constructed which approximates with order 8. Moreover, we show for a class of curves of non-zero measure that the optimal rate $2m$ is achieved.

[1]Jordan Uni. of Science and Technology, Mathematics, Irbid, 22110 Irbid, Jordan, rababah@just.edu.jo

Finite Element Approximation of Variably Saturated Subsurface Flow with Reactive Transport: Part II

Florin Adrian Radu[1]

This talk continues the previous contribution. The Richards' equation is now coupled with a model for reactive solute transport. In particular, biodegradation of contaminants is considered which is described by monod kinetics. The finite element discretization techniques used for this set of coupled convection-diffusion-reaction equations and the algorithmic solution process of the resulting nonlinear algebraic equations are presented. Another problem class being introduced is the coupled (un-)saturated flow and surfactant transport. Here, the Richards' equation and a transport equation for the surfactant are solved simultaneously. For both equations mixed finite elements discretizations are used. Finally, illustrative numerical studies are presented for realistic contaminated lands.

[1]University of Erlangen-Nuremberg, Chair of Applied Mathematics I, Martensstrasse 3, 91058 Erlangen, Germany,
raduf@am.uni-erlangen.de

Parallel and adaptive finite elements for the Navier-Stokes equation

Thomas Richter[1]

We present a parallel multigrid solver for partial differential equations. Based on powerful and stable sequential algorithms we receive a reliable code for solving various types of differential equations (including Navier Stokes, reactive flows, optimization problems, ...) on clusters.

The advantage of parallel computers is combined with a posteriori error estimation and functional orientated mesh adaption.

The vital goal of parallel adaption was closeness to the sequential code. Numerical examples demonstrate the versatility and efficiency of the considered methods.

References:

[1] Parallel and adaptive finite elements for the Navier-Stokes equation

[1]Universität Heidelberg, Institut für Angewandte Mathematik,
thomas.richter@iwr.uni-heidelberg.de

Anisotropic Mesh Generation and Mesh Refinement with Netgen

Joachim Schöberl[1]

Many real life problems contain anisotropic sub-structures such as thin shields or narrow holes. Usually in 3D, only anisotropic meshes for these structures make the finite element simulation possible. In the first part of the talk, we discuss the approach chosen in the mesh generator Netgen for defining and meshing of anisotropic domains.

The second part deals with aligned anisotropic refinement of given, isotropic or anisotropic meshes to capture the behavior of solutions. We discuss the principles of anisotropic multigrid methods. Several examples from magnetic field simulation are demonstrated.

[1]Department of Computational Mathematics, Johannes Kepler University Linz, Austria, js@jku.at

A robust boundary element method for nearly incompressible linear elasticity

Olaf Steinbach[1]

For boundary value problems in linear elasticity we consider a boundary element method which is robust for nearly incompressible materials. Based on the spectral properties of the single layer potential for the Stokes problem we introduce an orthogonal splitting of the trial space. The resulting variational problem is then well conditioned and can be discretized by using standard boundary element methods.

References:

[1] O. Steinbach, A robust boundary element method for nearly incompressible linear elasticity. Bericht 2002/05, SFB 404, 2002.

[1]Mathematisches Institut A, Universität Stuttgart, Pfaffenwaldirng 57, 70569 Stuttgart, steinbach@mathematik.uni-stuttgart.de

Adaptive solution of operator equations using wavelet frames

Rob Stevenson[1]

In their paper "Adaptive Wavelet Methods II - Beyond the Elliptic Case", Cohen, Dahmen and DeVore proposed an adaptive wavelet algorithm for solving operator equations, as differential or singular integral equations in variational form. Assuming that the operator defines a boundedly invertible mapping between a Hilbert space and its dual, and that a *Riesz basis* of wavelet type for this Hilbert space is available, the operator equation can be transformed into an equivalent well-posed infinite matrix-vector system. This system is solved by an iterative method, where each application of the infinite stiffness matrix is replaced by an adaptive approximation. Assuming that the stiffness matrix is sufficiently compressible, i.e., that it can sufficiently well be approximated by sparse matrices, it was proven that the adaptive method has optimal computational complexity in the sense that it converges with the same rate as the best N-term approximations for the solution assuming it would be explicitly available.

The condition concerning compressibility requires that, dependent on their order, the wavelets have sufficiently many vanishing moments, and that they are sufficiently smooth. Yet, except on tensor product domains, wavelets that satisfy this smoothness requirement are difficult to construct. We present an approach where the domain or manifold on which the operator equation is posed as an *overlapping* union of subdomains, each of them being the image under a smooth parametrization of the hypercube. By lifting wavelets on the hypercube to the the subdomains we obtain a *frame* for the Hilbert space. With this frame the operator equation is transformed into a matrix-vector system, after which this system is solved iteratively by an adaptive method similar to the one from Cohen, Dahmen and DeVore. With this approach, frame elements that have sufficiently many vanishing moments and that are sufficiently smooth, which is needed for the compressibility, are easily constructed. By handling additional difficulties due to the fact that a frame gives rise to an underdetermined matrix-vector system, we prove that this adaptive method has optimal computational complexity.

References:

[1] preprint, see my homepage

[1]University of Utrecht, Department of Mathematics, P.O. Box 80.010, NL-3508 TA Utrecht, The Netherlands ,
stevenso@math.uu.nl

Error bounds for Finite Element Solutions of Elliptic Variational Inequalities of second kind

Franz-Theo Suttmeier[1]

In this talk, we extend our studies on finite element Galerkin schemes for elliptic variational inequalities of first to the one of second kind. Especially we perfom the corresponding a posteriori error analysis for a model of the flow of a Bingham fluid and a friction problem.

References:

[1] Error bounds for Finite Element Solutions of Elliptic Variational Inequalities of second kind. EAST-WEST J. Numer. Math., Vol.9, No.4

[1]Universität Dortmund, Lehrstuhl für Wissenschaftliches Rechnen, Vogelpothsweg 87, 44221 Dortmund,
suttmeier@math.uni-dortmund.de

Projection methods for contact problems in elasticity

Roman Unger[1] Arnd Meyer[2]

The aim of the talk is showing, how projection methods can used for computing contact-problems in elasticity for different obstacles.

Starting with the projection idea for handling hanging nodes in finite element discretizations the extension of the method for handling penetrated nodes in contact problems will be described.

[1]TU Chemnitz, Fakultät für Mathematik , 09107 Chemnitz, Germany,
roman.unger@mathematik.tu-chemnitz.de

[2]TU Chemnitz, Fakultät für Mathematik , 09107 Chemnitz, Germany,
arnd.meyer@mathematik.tu-chemnitz.de

Adaptive Finite Element Methods for Parameter Identification Problems

Boris Vexler[1]

We consider parameter identification problems involving partial differential equ ations with finite number of unknown parameters.
Finite elements on locally refined meshes are employed for discretization of the state equation. An a posteriori error estimator for the error in the parameter is derived. It is used for quantitative error control and for successive improve ment of the accuracy by appropriate mesh refinement.
Numerical examples illustrate the behavior of the method.

[1]Institute of Applied Mathematics, University of Heidelberg,
Boris.Vexler@iwr.uni-heidelberg.de

Surname, first name, acad. title	**from**		**e-mail**
Apel, Thomas, Dr.	Chemnitz		apel@mathematik.tu-chemnitz.de
Bause, Markus, Dr.	Erlangen	Deutschland	bause@am.uni-erlangen.de
Beuchler, Sven, Dipl.-Math.	Chemnitz	Germany	sven.beuchler@mathematik.tu-chemnitz.de
Blum, Heribert, Prof	Dortmund	Germany	Heribert.Blum@Math.Uni-Dortmund.DE
Braack, Malte, Dr	Heidelberg		malte.braack@iwr.uni-heidelberg.de
Braess, Dietrich, Prof. Dr.	Bochum	Germany	braess@num.ruhr-uni-bochum.de
Dahmen, Wolfgang, Prof. Dr.	Aachen	Germany	dahmen@igpm.rwth-aachen.de
Derezine, Sviatoslav, Dipl.Math.	Chemnitz		derezine@mathematik.tu-chemnitz.de
Dolejsi, Vit, Dr.	Prague	Czech Republic	dolejsi@karlin.mff.cuni.cz
Eibner, Tino, Dipl.-Math.techn	Chemnitz	Germany	teibner@mathematik.tu-chemnitz.de
Flemisch, Bernd, M.Sc.	Stuttgart	Germany	flemisch@mathematik.uni-stuttgart.de
Formaggia, Luca, Prof.	Milano	Italy	luca.formaggia@epfl.ch
Grosman, Sergej, M.Sc.	Chemnitz		grosman@mathematik.tu-chemnitz.de
Harbrecht, Helmut, Dr.	Chemnitz		harbrech@mathematik.tu-chemnitz.de
Heinrich, Bernd, Prof. Dr.	Chemnitz	Germany	heinrich@mathematik.tu-chemnitz.de
Heuveline, Vincent, Dr.	Heidelberg	Germany	vincent.heuveline@iwr.uni-heidelberg.de
Hülsemann, Frank, PhD	Erlangen	Germany	frank.huelsemann@cs.fau.de
Jung, Michael, Doz. Dr.	Dresden	BRD	mjung@math.tu-dresden.de
Kienesberger, Johanna, Dipl.-Ing.	Linz	Austria	johanna.kienesberger@jku.at
Kunert, Gerd, Dr.	Chemnitz	Germany	gerd.kunert@mathematik.tu-chemnitz.de
Lamichhane, Bishnu Prasad, M. Sc.	Stuttgart	Germany	lamichhane@mathematik.uni-stuttgart.de
Li , Yonghai , Dr.	Beijing	China	yonghai-li@163.com
Lube, Gert, Prof. Dr.	Goettingen	Germany	lube@math.uni-goettingen.de
Maischak, Matthias, PD Dr.	Chemnitz	Germany	maischak@ifam.uni-hannover.de
Matthies, Gunar, Dipl.-Math.	Magdeburg	Germany	matthies@mathematik.uni-magdeburg.de

Surname, first name, acad. title	**from**		**e-mail**
Meyer, Arnd, Prof	Chemnitz	Germany	a.meyer@mathematik.tu-chemnitz.de
Micheletti, Stefano, Dr.	Milano	Italy	mike@mate.polimi.it
Pester, Cornelia, Dipl-Math	Chemnitz		cornelia.pester@mathematik.tu-chemnitz.de
Pester, Matthias, Dr	Chemnitz		pester@mathematik.tu-chemnitz.de
Pietsch, Kornelia, Dipl.-Math. techn.	Chemnitz		kornelia.pietsch@hrz.tu-chemnitz.de
Rababah, Abedallah, Prof	Irbid	Jordan	rababah@just.edu.jo
Radu, Florin Adrian, Dipl.Math.	Erlangen	Germany	raduf@am.uni-erlangen.de
Richter, Thomas, Dipl. Math	Heidelberg		thomas.richter@iwr.uni-heidelberg.de
Schneider, Reinhold, Prof.	Chemnitz		reinhold@mathematik.tu-chemnitz.de
Schneider, Rene,	Schoenerstadt		rene.schneider@hrz.tu-chemnitz.de
Schöberl, Joachim, Dr.	Linz	Austria	js@jku.at
Smuglyakov, Alexander, Dipl.	Chemnitz	Deutschland	alsm@mathematik.tu-chemnitz.de
Solovyev, Sergey, Dr.	Chemnitz		solovev@mathematik.tu-chemnitz.de
Steidten, Torsten, Dipl.-Math.	Gelenau		Torsten.Steidten@mathematik.tu-chemnitz.de
Steinbach, Olaf, PD Dr	Stuttgart	Germany	steinbach@mathematik.uni-stuttgart.de
Stevenson, Rob, Dr.	Utrecht	The Netherlands	stevenso@math.uu.nl
Suttmeier, Franz-Theo, Dr. habil.	Dortmund		suttmeier@math.uni-dortmund.de
Unger, Roman, Dipl.-Math. techn.	Chemnitz	Germany	roman.unger@mathematik.tu-chemnitz.de
Vexler, Boris, Dipl.-Math.	Heidelberg	Germany	Boris.Vexler@iwr.uni-heidelberg.de
Winkler, Gunter, Dipl. math. oec.	Chemnitz	Germany	gunter.winkler@mathematik.tu-chemnitz.de