

An introduction to optimal control of partial differential equations

Fredi Tröltzsch, TU Berlin

This short course introduces to the numerical analysis of optimal control problems for partial differential equations (PDEs) with special emphasis on some nonlinear reaction diffusion equations, namely the Nagumo and FitzHugh-Nagumo equations. This class of PDEs exhibits traveling wave fronts or spiral waves as typical solutions, and there are impressive examples for controlling them.

Main issues of optimal control theory will be addressed: Existence of optimal controls, differentiability of the control-to-state mapping, necessary optimality conditions and adjoint equation, up to elements of the subdifferential calculus of convex optimization. Finally, it will be demonstrated how traveling or spiral waves can be moved in an optimal way by sparse controls.

The course starts with simple examples of control problems for ordinary differential equations and PDEs. Main ideas are explained first in a finite-dimensional setting to avoid the technicalities that are characteristic for the analysis of PDEs. Later, the results will be rigorously formulated in appropriate function spaces for weak solution of PDEs. The course will proceed from the control of the Poisson equation via the nonlinear heat equation up to the level of the FitzHugh-Nagumo system. Various examples will illustrate the findings.