Necessity, Possibility and Likelihood in Syllogistic Reasoning

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Syllogistic Reasoning

All A are B
Some B are C

What follows / is possible / is likely for A and C?

- A syllogism consists of **two quantified statements** (premises) which **interrelate** three terms (A, B, C)
- Usually, the task is to conclude what **necessarily** follows

- Few studies about how humans infer **possible** conclusions [1]
- We investigated the differences between finding **necessary**, **possible** and **likely** conclusions – an important everyday reasoning task for humans [2]

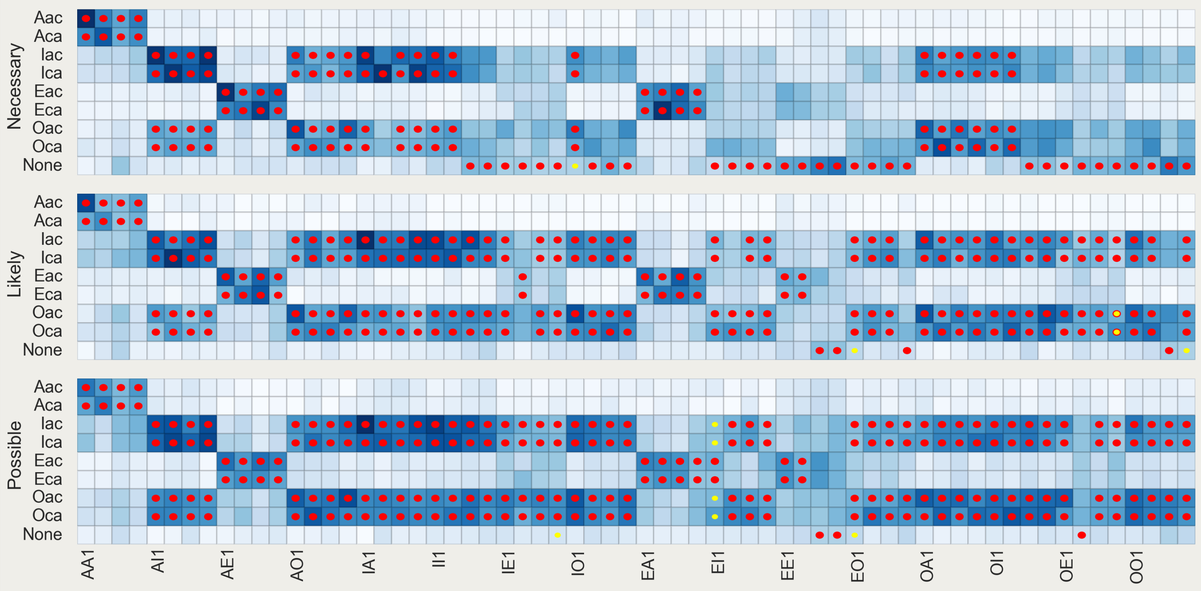
Effects of Figure and Quantifiers

- **Figural effect** [4] was significant for *Necessary* and *Likely*, but not for *Possible*
- The figural effect was strongest for *Likely*, indicating that it is not only an effect for logical reasoning, but also a **preference effect**

Quant	Nec.	Likely	Poss.
A → I	.232	.277	.605
I → A	.047	.033	.133
E → O	.088	.141	.461
O → E	.050	.067	.231

- Quantifier co-occurrence indicates a **pragmatic interpretation** [5]
- For *Possible*, universal **quantifiers entail particular quantifiers**, but not the inverse
- Results for *Necessary* and *Likely* are **nearly identical**

Response Pattern Analysis



- Participants often consider **multiple responses** as correct highlighting the importance of the multiple-choice response format [3]
- Visually, *Likely* appears to be in between *Necessary* and *Possible*

Data 1	Data 2	RMSE	MFA
Nec.	Likely	.093	.780
Nec.	Poss.	.154	.765
Poss.	Likely	.101	.940

- *Likely* and *Possible* have nearly identical patterns of the most frequently selected answer combinations (MFA)
- For answer distributions, *Likely* and *Necessary* are most similar (RMSE)

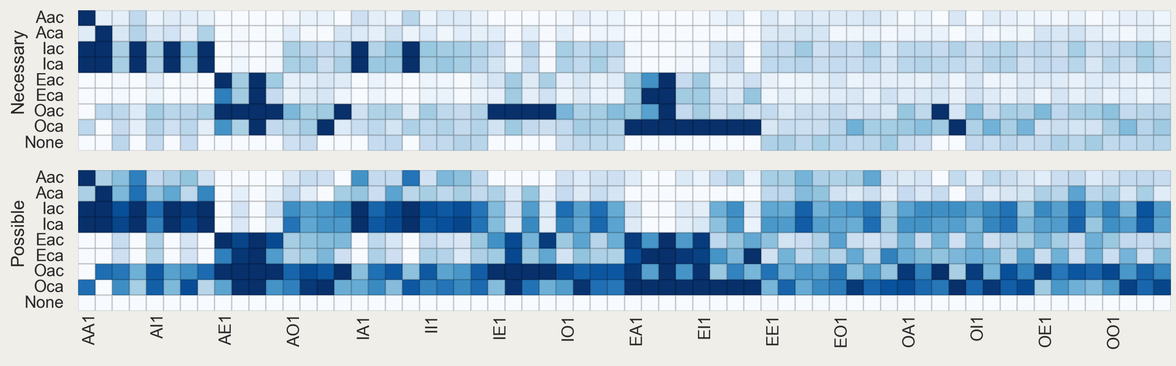
- The biggest difference lies in the frequency of participants not selecting any responses (*None*)
- In line with the logically correct responses, ***None* is most prominent for *Necessary***

Dataset	Accuracy	Precision	Recall	Specificity
Necessary	.767	.286	.486	.803
Possible	.416	.981	.363	.925

- Participants were good at **avoiding selecting incorrect conclusions** at the expense of missing correct ones

→ For Necessary (few correct conclusions) this results in a high accuracy
→ For Possible (many correct conclusions) this results in a high precision

Model Analysis



- **mReasoner** was used as an implementation of the Mental Model Theory that is able to predict responses for *Necessary* and *Possible* [6]
- We used the **best fit** of mReasoner for each **individual participant** to generate the patterns
- Patterns for *Necessary* and *Possible* **did not capture** participants' behaviour convincingly

Data	RMSE	MFA
Nec.	.212	.776
Poss.	.253	.777

Dataset	Accuracy	Precision	Recall	Specificity
Necessary (mR)	0.834	0.499	0.987	.817
Possible (mR)	0.661	1	0.626	1

- mReasoner **overestimated the logical correctness** of participants
- Instead, participants seemed to rely on **pragmatic interpretations** of quantifiers

Conclusion

- **First dataset** with individual patterns for *necessary*, *possible* and *likely*.
- *Likely* appears to be **a middle ground** between *possible* and *necessary*
- Well-suited to investigate **biases** and **preferences** in human reasoning
- Currently, there is **no model** that can explain or capture these patterns sufficiently
- **Figures of premises** interpreted as implicitly hinting at some conclusions making them appear more likely
- Logical correctness mostly influenced by different **interpretations of quantifiers**

References

[1] Evans, J., Handley, S., Harper, C., & Johnson-Laird, P. (1999). Reasoning about necessity and possibility: A test of the mental model theory of deduction. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 25, 1495-1513.

[2] Ragni, M., & Johnson-Laird, P. N. (2020). Reasoning about epistemic possibilities. *Acta Psychologica*, 208, 103081.

[3] Brand, D., & Ragni, M. (2023). Effect of response format on syllogistic reasoning. In M. Goldwater, F. K. Anggoro, B. K. Hayes, & D. C. Ong (Eds.), *Proceedings of the 45th Annual Conference of the Cognitive Science Society* (pp. 2408–2414).

[4] Dickstein, L. S. (1978). The effect of figure on syllogistic reasoning. *Memory & Cognition*, 6, 76–83.

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