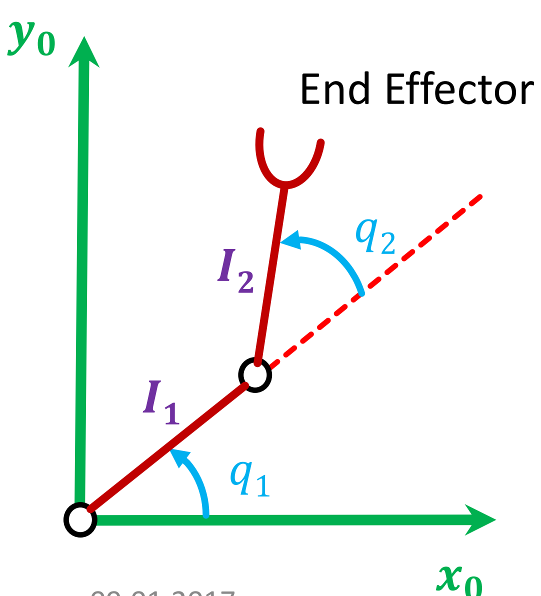
Example: RR Robot

Forward kinematics:

$$X_e = l_1 c_1 + l_2 c_{12}$$
$$Y_e = l_1 s_1 + l_2 s_{12}$$



$$\mathcal{J}_1 = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} \end{bmatrix}, \mathcal{J}_2 = \begin{bmatrix} -l_2 s_{12} \\ l_2 c_{12} \end{bmatrix}$$

Illustrate the column vector of the Jacobian in the space at the end-effector point.

$\mathcal{J}_2$ points in the direction perpendicular to link 2.

While $\mathcal{J}_1$ is not perpendicular to link 1 but is perpendicular to the vector (X_e,Y_e).
This is because $\mathcal{J}_1$ represent the endpoint velocity caused by joint 1 when joint 2 is not rotating.
In other word, link 1 and 2 are rigidly connected, becoming a single rigid body of length (X_e,Y_e) and $\mathcal{J}_1$ is the tip velocity of this body.

Example: RR Robot

Forward kinematics:

$$X_e = I_1 c_1 + I_2 c_{12}$$
$$Y_e = I_1 s_1 + I_2 s_{12}$$

$$\mathcal{J}_1 = \begin{bmatrix} -I_1 s_1 - I_2 s_{12} \\ I_1 c_1 + I_2 c_{12} \end{bmatrix}, \mathcal{J}_2 = \begin{bmatrix} -I_2 s_{12} \\ I_2 c_{12} \end{bmatrix}$$

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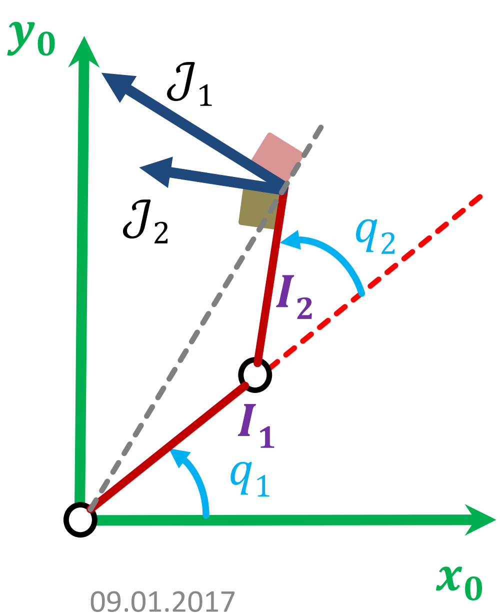


Example: RR Robot

Forward kinematics:

$$X_e = l_1 c_1 + l_2 c_{12}$$
$$Y_e = l_1 s_1 + l_2 s_{12}$$

$$\mathcal{J}_1 = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} \end{bmatrix}, \mathcal{J}_2 = \begin{bmatrix} -l_2 s_{12} \\ l_2 c_{12} \end{bmatrix}$$

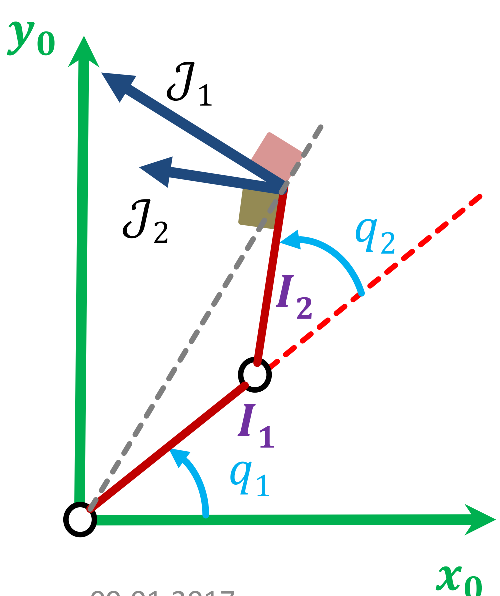
If the two Jacobian column vectors **are aligned**, the end-effector can not be moved in an arbitrary direction.

This may happen for particular arm configurations when the two links are fully contracted or extracted.

These arm configurations are referred to as singular configurations.

ACCORDINGLY, the Jacobian matrix become singular at these positions.

Find out the singular configurations…

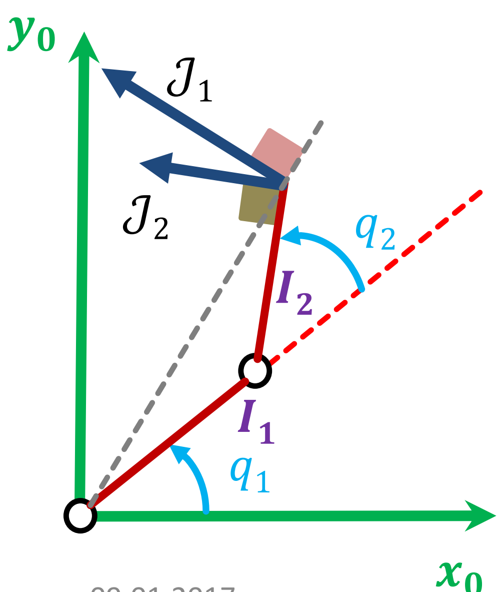


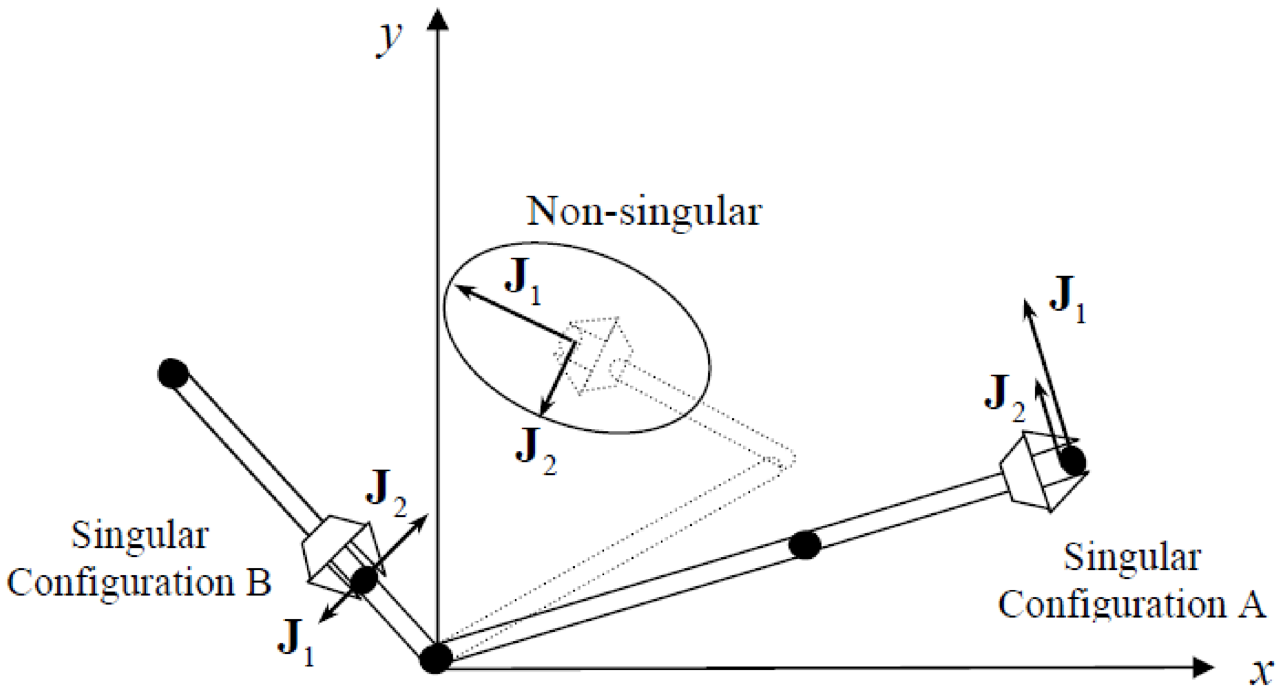
Example: RR Robot

Forward kinematics:

$$\mathcal{J}_1 = \begin{bmatrix} -I_1 s_1 - I_2 s_{12} \\ I_1 c_1 + I_2 c_{12} \end{bmatrix}, \mathcal{J}_2 = \begin{bmatrix} -I_2 s_{12} \\ I_2 c_{12} \end{bmatrix}$$

$$X_e = I_1\, c_1 + I_2\, c_{12}$$
$$Y_e = I_1\, s_1 + I_2\, s_{12}$$





Example: RR Robot

Forward kinematics:

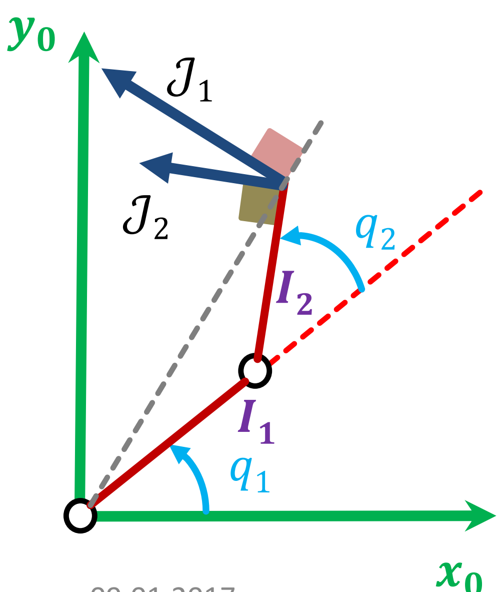
$$X_e = I_1\, c_1 + I_2\, c_{12}$$
$$Y_e = I_1\, s_1 + I_2\, s_{12}$$

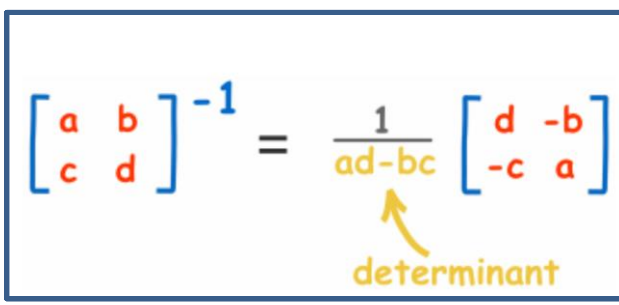
$$\mathcal{J}_v = \begin{bmatrix} -I_1 s_1 - I_2 s_{12} & -I_2 s_{12} \\ I_1 c_1 + I_2 c_{12} & I_2 c_{12} \end{bmatrix}$$

The Jacobian reflects the singular configurations.

When joint 2 is 0 or 180 degrees:

$$det(\mathcal{J}_v) = \det\left(\quad \right) =$$





Example: RR Robot

Forward kinematics:

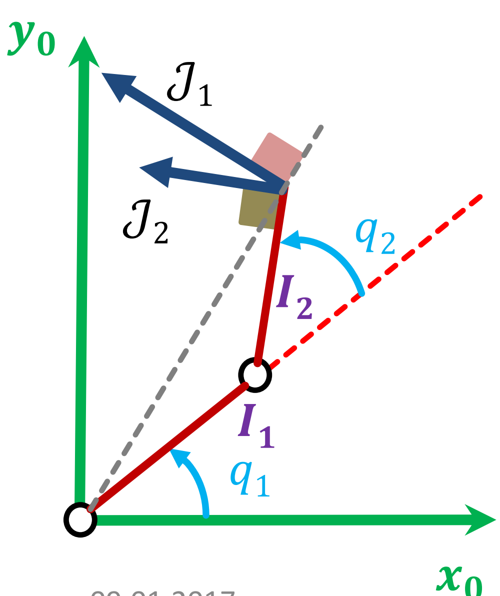
$$X_e = I_1\, c_1 + I_2\, c_{12}$$
$$Y_e = I_1\, s_1 + I_2\, s_{12}$$

$$\mathcal{J}_v = \begin{bmatrix} -I_1 s_1 - I_2 s_{12} & -I_2 s_{12} \\ I_1 c_1 + I_2 c_{12} & I_2 c_{12} \end{bmatrix}$$

The Jacobian reflects the singular configurations.

When joint 2 is 0 or 180 degrees:

$$det(\mathcal{J}_v) = \det\left(\begin{bmatrix} -(I_1 \pm I_2) s_1 & \mp I_2 s_1 \\ (I_1 \pm I_2) c_1 & \pm I_2 c_1 \end{bmatrix}\right) = 0$$



$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

determinant

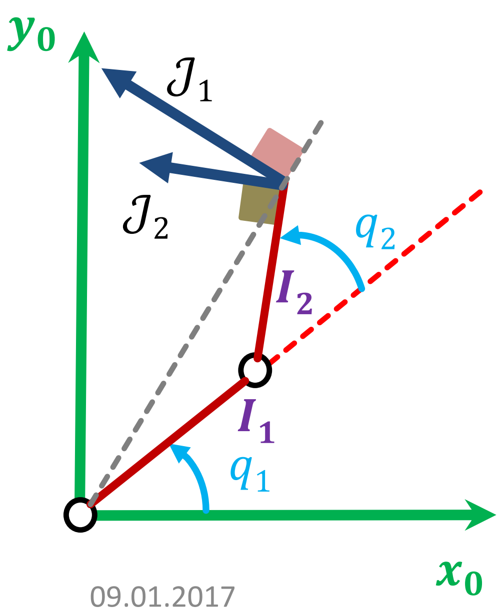
Example: RR Robot

Work out the joint velocities $\dot{q}$ ($\dot{q}_1$, $\dot{q}_2$) in terms of the end effector velocity $V_e(V_x, V_y)$.

If the arm configuration is not singular, this can be obtained by taking the inverse of the Jacobian matrix:

$$\dot{q} = J^{-1}.Ve$$

Note that the differential kinematics problem has **a unique solution** as long as the Jacobian is non-singular.

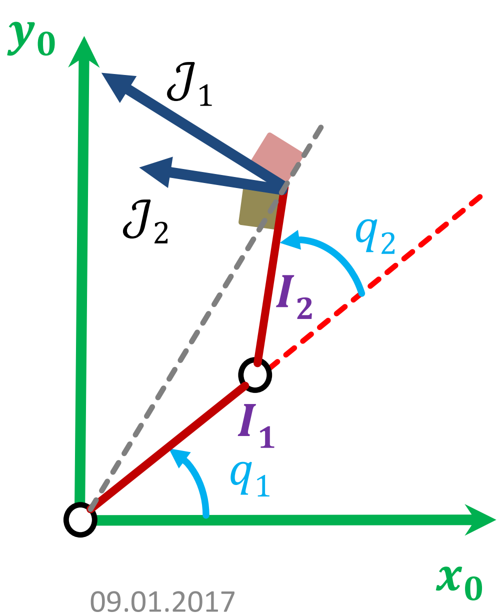


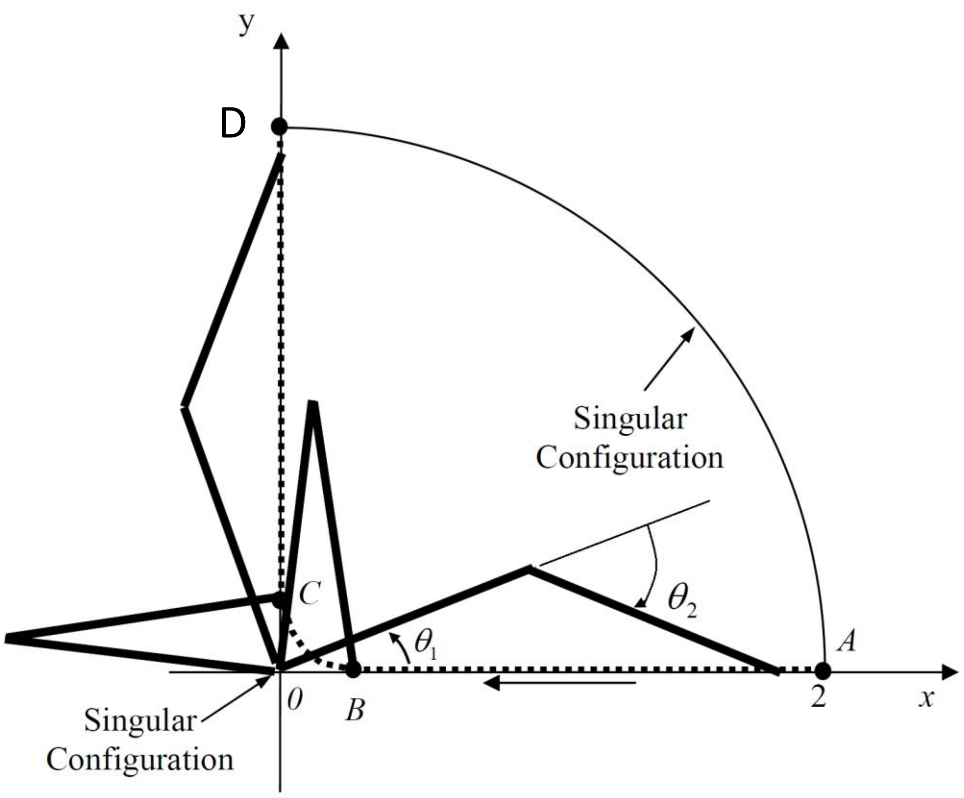
Since the elements of the Jacobian matrix are function of joint displacements, the inverse Jacobian varies depending on the arm configuration.
This means that **although the desired end-effector velocity is constant, the joint velocities are not.**

Example: RR Robot

We want to move the endpoint of the robot at a constant speed along a path starting at point "A" on the x-axis, (+2.0, 0), go around the origin through point "B" (+ε, 0) and "C" (0, +ε), and reach the final point "D" (0, +2.0) on the y-axis. Consider $l_1 = l_2$.

Work out joints velocities along this path.





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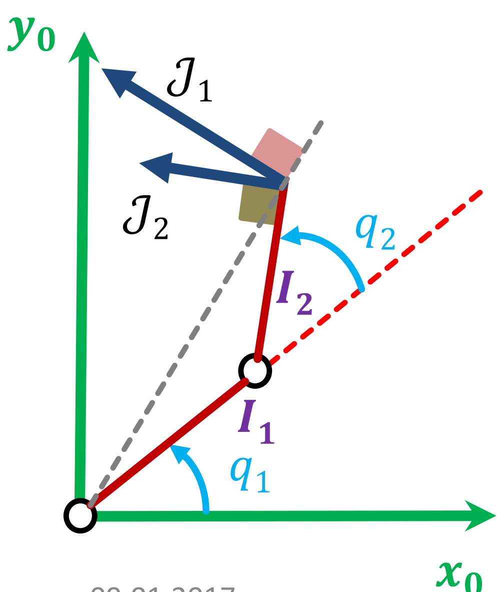
Work out joints velocities along this path.

The Jacobian is:

$$\mathcal{J}_v = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix}$$



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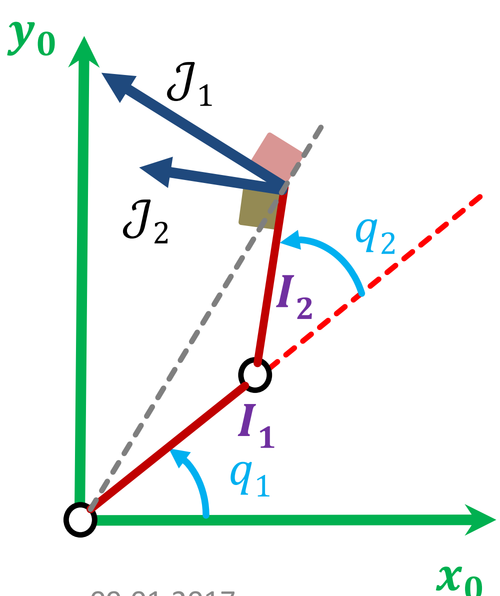
Work out joints velocities along this path.



The Jacobian is:

$$\mathcal{J}_v = \begin{bmatrix} -I_1 s_1 - I_2 s_{12} & -I_2 s_{12} \\ I_1 c_1 + I_2 c_{12} & I_2 c_{12} \end{bmatrix}$$

The inverse of the Jacobian is:

$$\mathcal{J}_v^{-1} = \frac{1}{\quad}\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

determinant

Example: RR Robot

We want to move the endpoint of the robot at a constant speed along a path starting at point "A" on the x-axis, (+2.0, 0), go around the origin through point "B" (+ε, 0) and "C" (0, +ε), and reach the final point "D" (0, +2.0) on the y-axis. Consider $I_1 = I_2$.

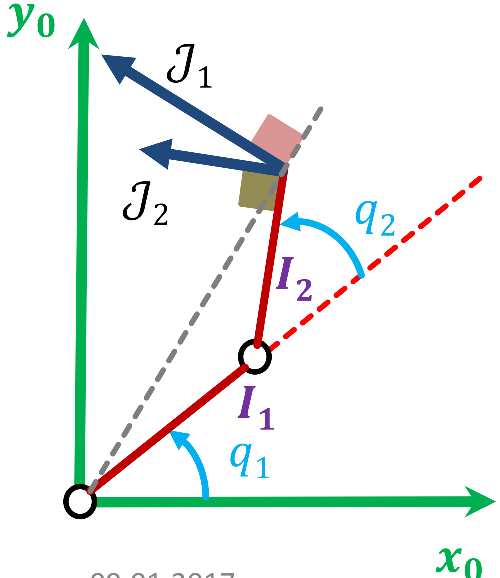
Work out joints velocities along this path.

The inverse of the Jacobian is:

$$\mathcal{J}_v^{-1} = \frac{1}{I_1 I_2 s_2} \begin{bmatrix} I_2 c_{12} & I_2 s_{12} \\ -I_1 c_1 - I_2 c_{12} & -I_1 s_1 - I_2 s_{12} \end{bmatrix}$$

$$\dot{q}_1 =$$

$$\dot{q}_2 =$$



TwoLinksManipulator2.m

```
% two links arm
clc;
clear
%%%%%%%%%%%%%%%
% A to B
t1_AB = 0.01 : 0.02 : ((pi/2.0)-0.01);
t2_AB = -2*t1_AB;

Vx_AB = -1.0*ones(size(t1_AB)) ; % [m/s]
Vy_AB =  0.0*ones(size(t1_AB)) ; % [m/s]
%%%%%%%%%%%%%
t1 = [t1_AB ]; t2 = [t2_AB ];
Vx = [Vx_AB ]; Vy = [Vy_AB ];

t1D = (Vx.*cos(t1+t2)+Vy.*sin(t1+t2))./sin(t2);
t2D = -(Vx.*(cos(t1)+cos(t1+t2))+Vy.*(sin(t1)+sin(t1+t2)))./sin(t2

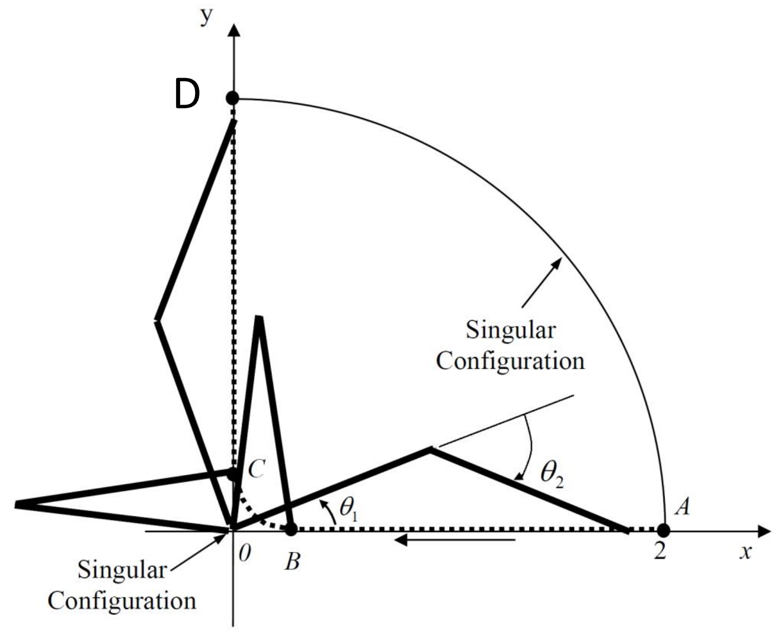
plot(t1D); hold on; plot(t2D);
%%%%%%%%%%%%%
% C to D
t1_CD = pi-0.01 : -0.02 : ((pi/2.0)+0.01);
t2_CD = t1_CD - pi/2.0 ;

Vx_CD =  0.0*ones(size(t1_AB)) ; % [m/s]
Vy_CD =  1.0*ones(size(t1_AB)) ; % [m/s]
%%%%%%%%%%%%%
t1 = [t1_CD ]; t2 = [t2_CD ];

Vx = [Vx_CD ]; Vy = [Vy_CD ];

t1D = (Vx.*cos(t1+t2)+Vy.*sin(t1+t2))./sin(t2);
t2D = -(Vx.*(cos(t1)+cos(t1+t2))+Vy.*(sin(t1)+sin(t1+t2)))./sin(t2);

plot(t1D); hold on; plot(t2D);
%%%%%%%%%%%%%
```



$$\dot{q}_1 = \frac{I_2 c_{12}.Vx + I_2 s_{12}.Vy}{I_1 I_2 s_2}$$

$$\dot{q}_2 = \frac{[-I_1 c_1 - I_2 c_{12}].Vx + [-I_1 s_1 - I_2 s_{12}].Vy}{I_1 I_2 s_2}$$

TwoLinksManipulator2.m

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clc;
clear
%%%%%%%%%%%%%%%
% A to B
t1_AB = 0.01
t2_AB = -2*t1_

Vx_AB = -1.0*
Vy_AB =  0.0*
%%%%%%%%%%%%%
t1 = [t1_AB ]
Vx = [Vx_AB ]

t1D = (Vx.*co
t2D = -(Vx.*(                t2)))./sin(t2

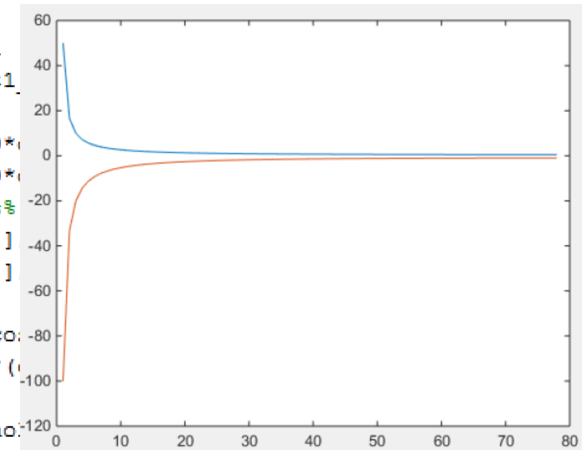
plot(t1D); ho
%%%%%%%%%%%%%
% C to D
t1_CD = pi-0.
t2_CD = t1_CD

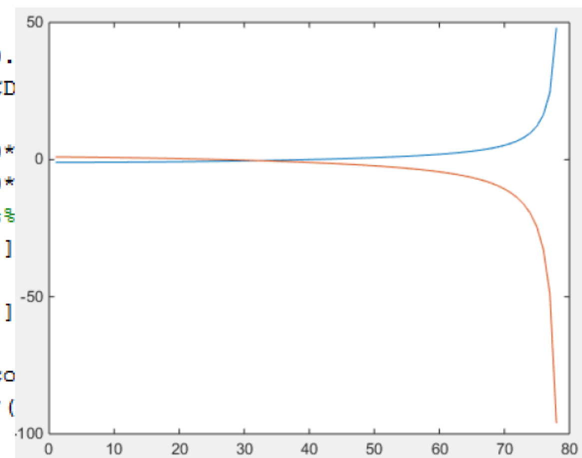
Vx_CD =  0.0*
Vy_CD =  1.0*
%%%%%%%%%%%%%
t1 = [t1_CD ]

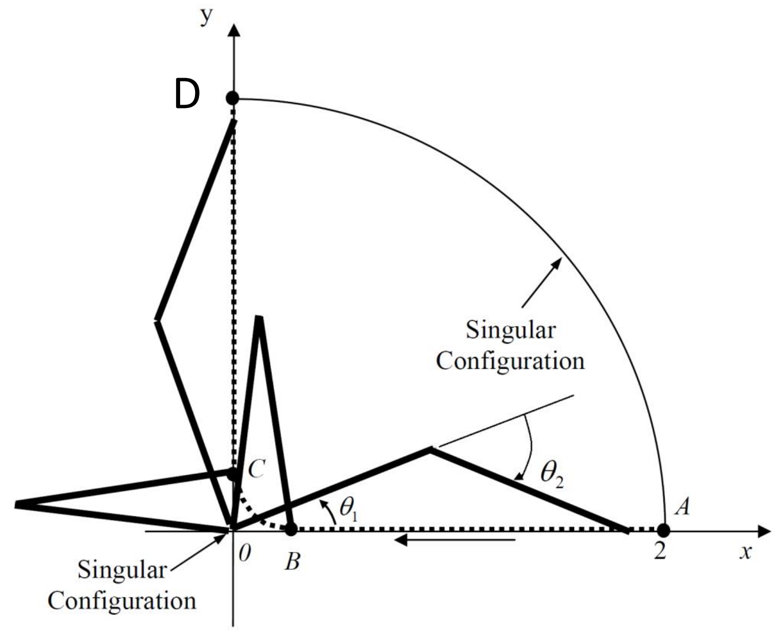
Vx = [Vx_CD ]

t1D = (Vx.*co
t2D = -(Vx.*(                t2)))./sin(t2);

plot(t1D); hold on; plot(t2D);
%%%%%%%%%%%%%
```







$$\dot{q}_1 = \frac{I_2 c_{12}.Vx + I_2 s_{12}.Vy}{I_1 I_2 s_2}$$

$$\dot{q}_2 = \frac{[-I_1 c_1 - I_2 c_{12}].Vx + [-I_1 s_1 - I_2 s_{12}].Vy}{I_1 I_2 s_2}$$

Example: RR Robot

$$\dot{q}_1 = \frac{I_2 c_{12}.Vx + I_2 s_{12}.Vy}{I_1 I_2 s_2}$$

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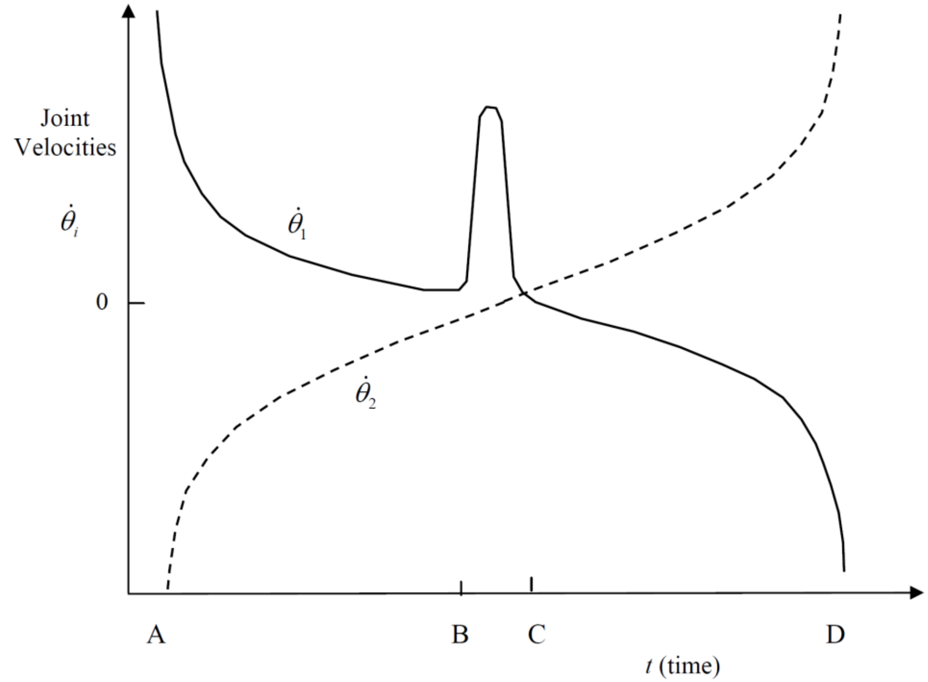
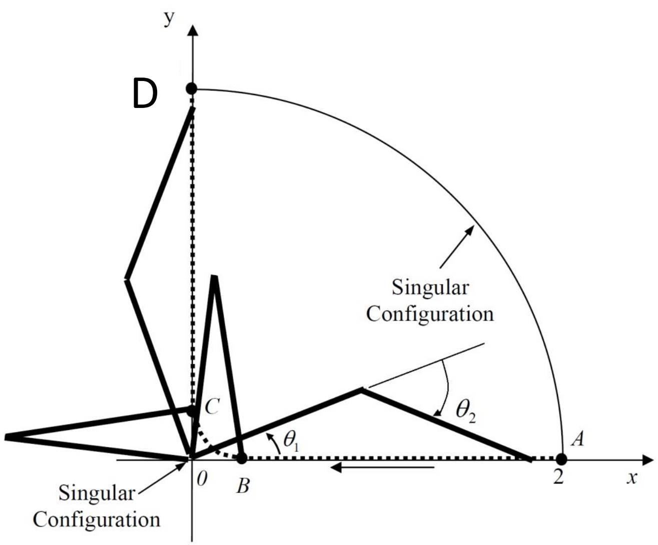


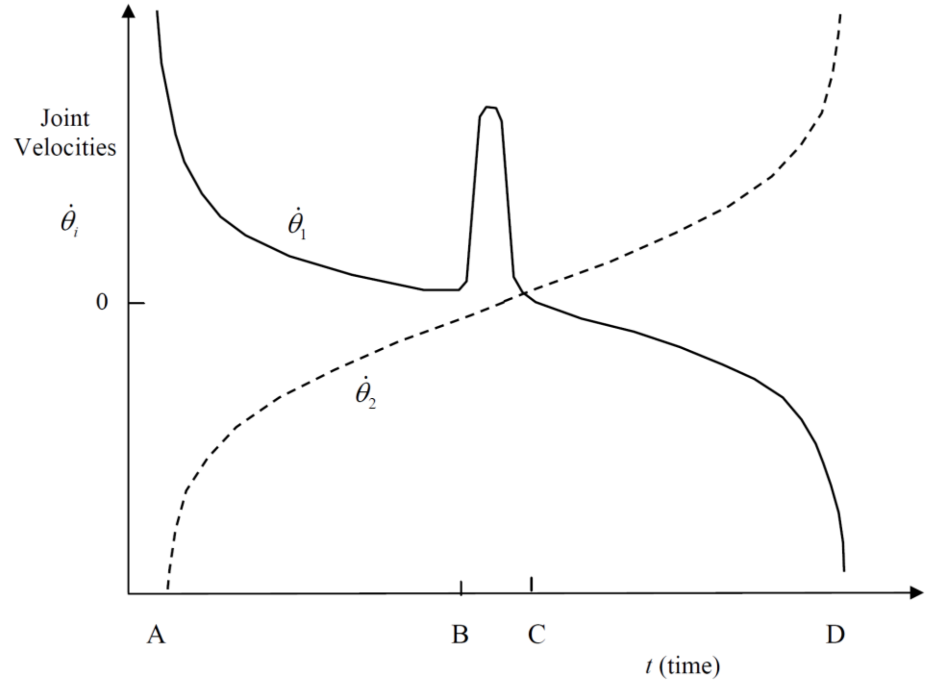


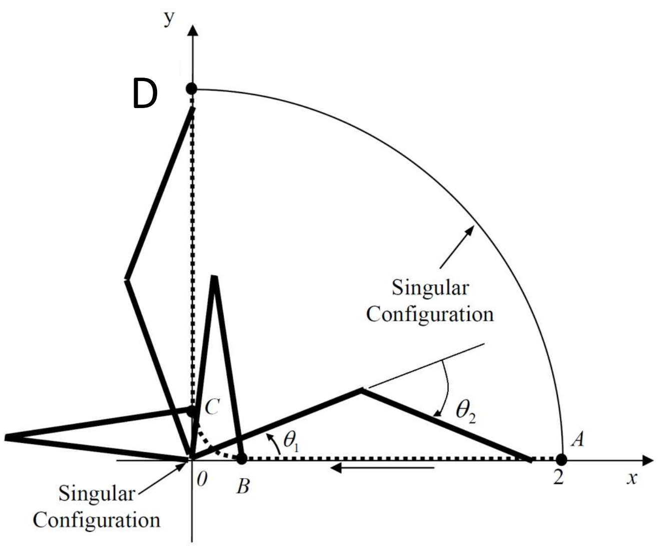
Note that the joint velocities are extremely large near the initial and the final points, and are unbounded at points A and D. These are the arm singular configurations q_2=0.

Example: RR Robot

$$\dot{q}_1 = \frac{l_2 c_{12}.Vx + l_2 s_{12}.Vy}{l_1 l_2 s_2}$$

$$\dot{q}_2 = \frac{[-l_1 c_1 - l_2 c_{12}].Vx + [-l_1 s_1 - l_2 s_{12}].Vy}{l_1 l_2 s_2}$$



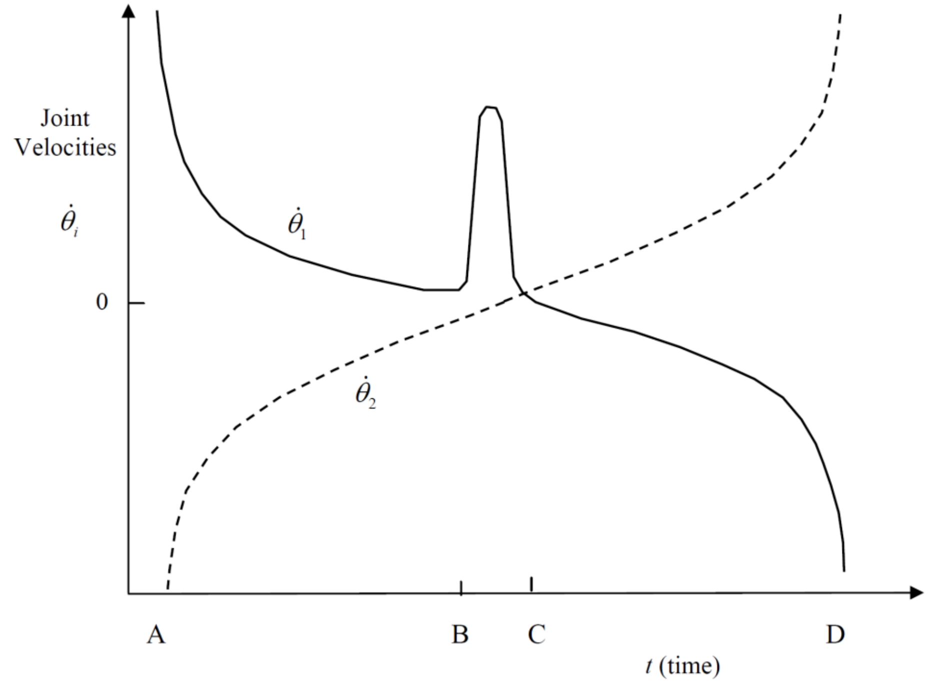


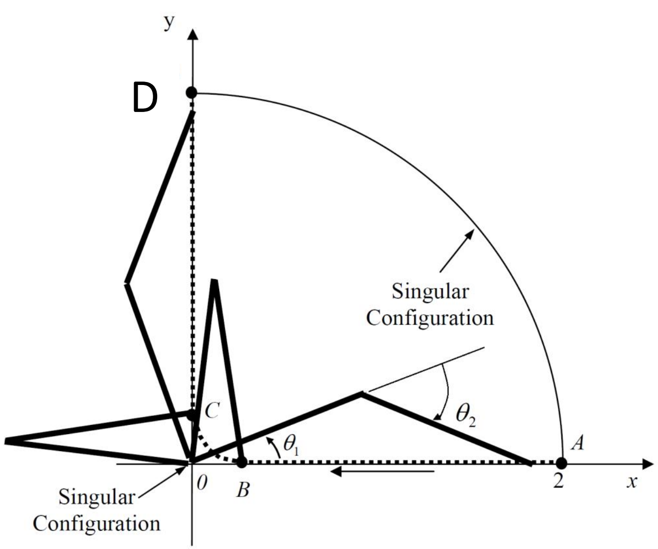
As the end-effector gets close to the origin, the velocity of the first joint becomes very large in order to quickly turn the arm around from point B to C. At these configurations, the second joint is almost -180 degrees, meaning that the arm is near singularity.

Example: RR Robot

$$\dot{q}_1 = \frac{I_2 c_{12}.Vx + I_2 s_{12}.Vy}{I_1 I_2 s_2}$$

$$\dot{q}_2 = \frac{[-I_1 c_1 - I_2 c_{12}].Vx + [-I_1 s_1 - I_2 s_{12}].Vy}{I_1 I_2 s_2}$$





This result agrees with the singularity condition using the determinant of the Jacobian:

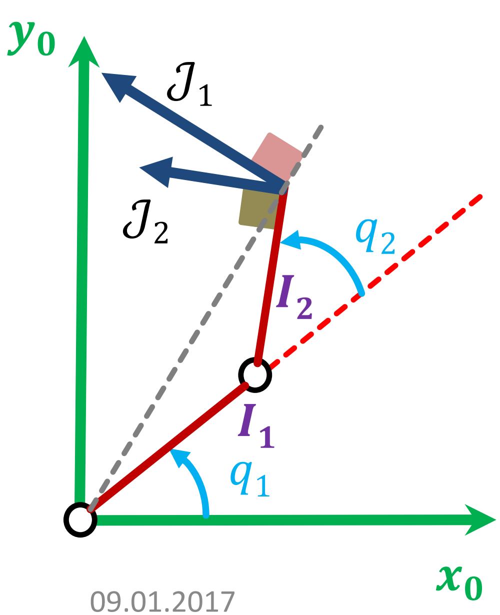
$det(\mathcal{J}_v) = \sin(q_2) = 0$ for $q_2 = k\pi, \ k = 0, \pm 1, \pm 2, \ldots$

Example: RR Robot

Using the Jacobian, analyse the arm behaviour at the singular points. Consider (l_1=l_2=1).

The Jacobian is:

$$\mathcal{J}_v = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix}, \mathcal{J}_1 = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} \end{bmatrix}, \mathcal{J}_2 = \begin{bmatrix} -l_2 s_{12} \\ l_2 c_{12} \end{bmatrix}$$



For q_2=0:

$$\mathcal{J}_1 = \begin{bmatrix} -2s_1 \\ 2c_1 \end{bmatrix}, \mathcal{J}_2 = \begin{bmatrix} -s_1 \\ c_1 \end{bmatrix}$$

The Jacobian column vectors reduce to the ones in the same direction.
Note that no endpoint velocity can be generated in the direction perpendicular to the aligned arm links (singular configuration A and D).

Example: RR Robot

Using the Jacobian, analyse the arm behaviour at the singular points. Consider (l_1=l_2=1).

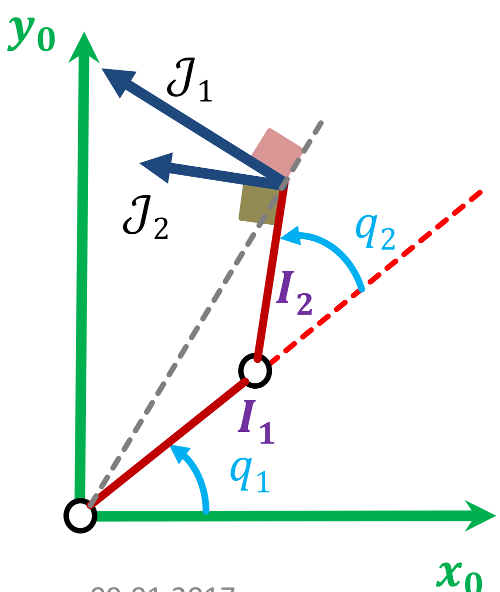
The Jacobian is:

$$\mathcal{J}_v = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix}, \mathcal{J}_1 = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} \end{bmatrix}, \mathcal{J}_2 = \begin{bmatrix} -l_2 s_{12} \\ l_2 c_{12} \end{bmatrix}$$

For $q_2=\pi$:

$$\mathcal{J}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathcal{J}_2 = \begin{bmatrix} s_1 \\ -c_1 \end{bmatrix}$$

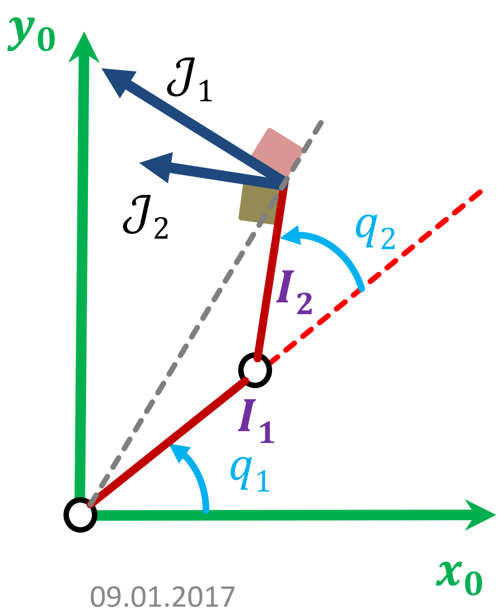
The first joint cannot generate any endpoint velocity, since the arm is fully contracted (singular configuration B).



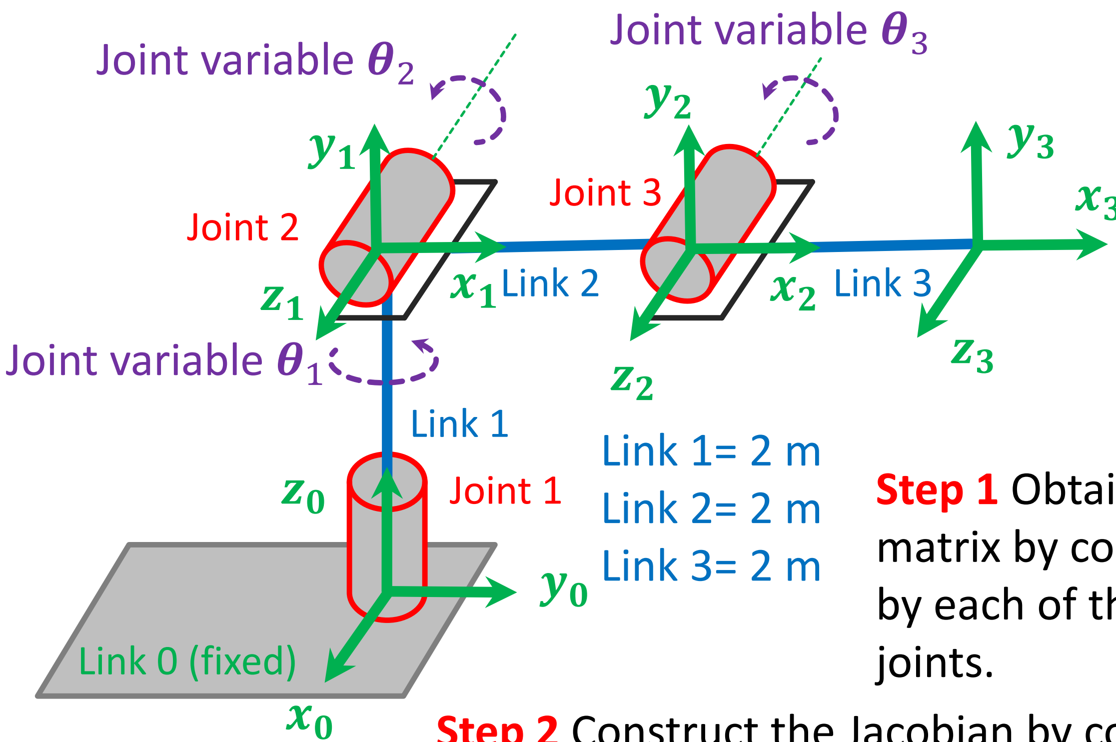
Example: RR Robot

Using the Jacobian, analyse the arm behaviour at the singular points. Consider (l_1=l_2=1).

At the singular configuration, there is at least one direction is which the robot cannot generate a non-zero velocity at the end effector.



Example: RRR Robot



The robot has three revolute joints that allow the endpoint to move in the three dimensional space. However, this robot mechanism has singular points inside the workspace. Analyze the singularity, following the procedure below.

Step 1 Obtain each column vector of the Jacobian matrix by considering the endpoint velocity created by each of the joints while immobilizing the other joints.

Step 2 Construct the Jacobian by concatenating the column vectors, and set the determinant of the Jacobian to zero for singularity: det J =0.

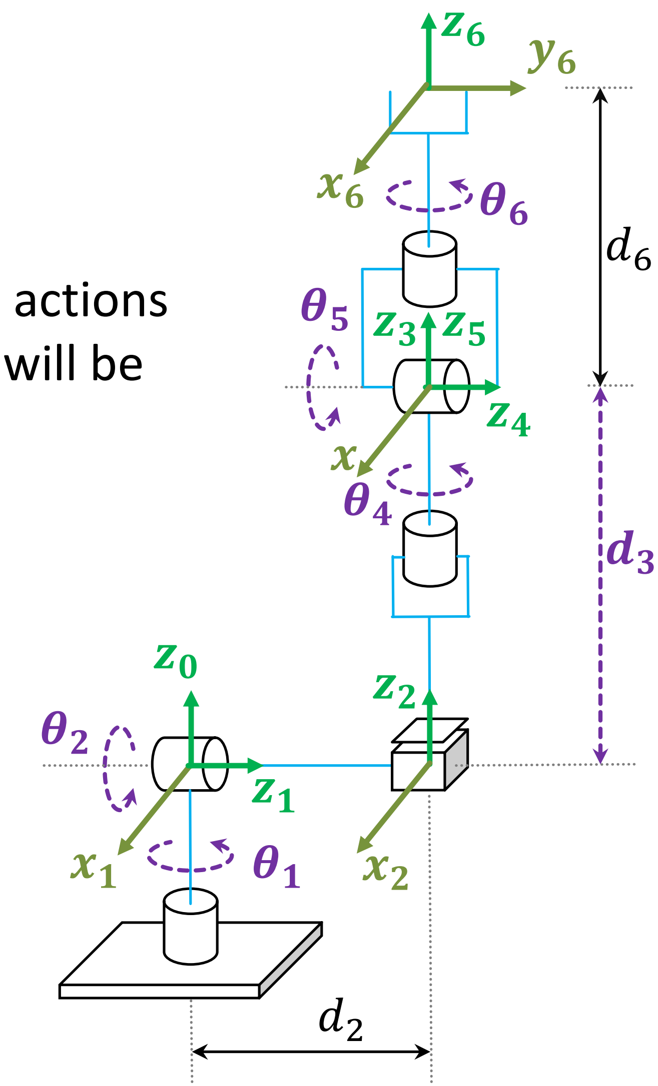
Step 3 Find the joint angles that make det J =0.

Step 4 Show the arm posture that is singular. Show where in the workspace it becomes singular. For each singular configuration, also show in which direction the endpoint cannot have a non-zero velocity.

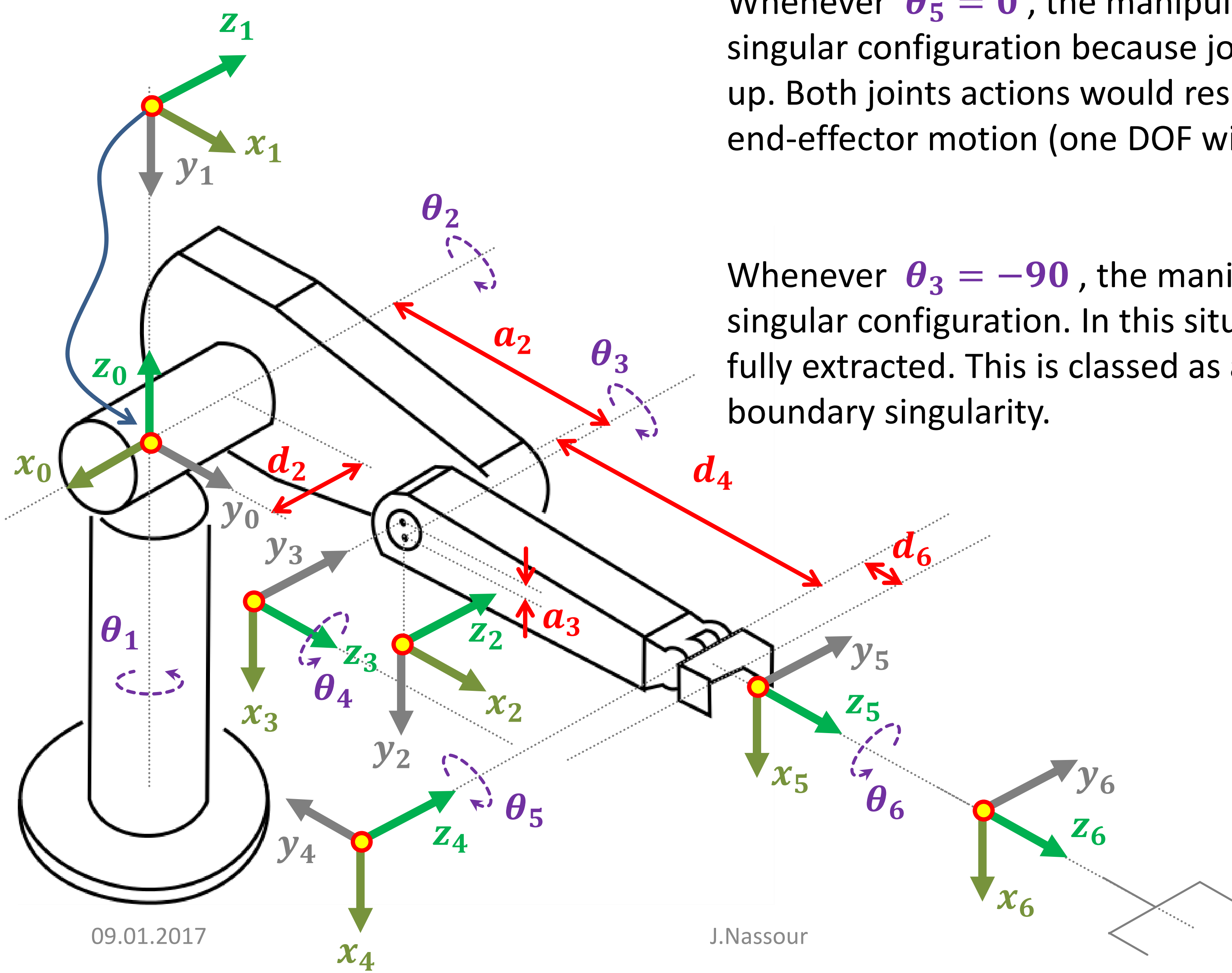
Stanford Arm

Give one example of singularity that can occur.

Whenever $\boldsymbol{\theta_5 = 0}$, the manipulator is in a singular configuration because joint 4 and 6 line up. Both joints actions would results the same end-effector motion (one DOF will be lost).



PUMA 260

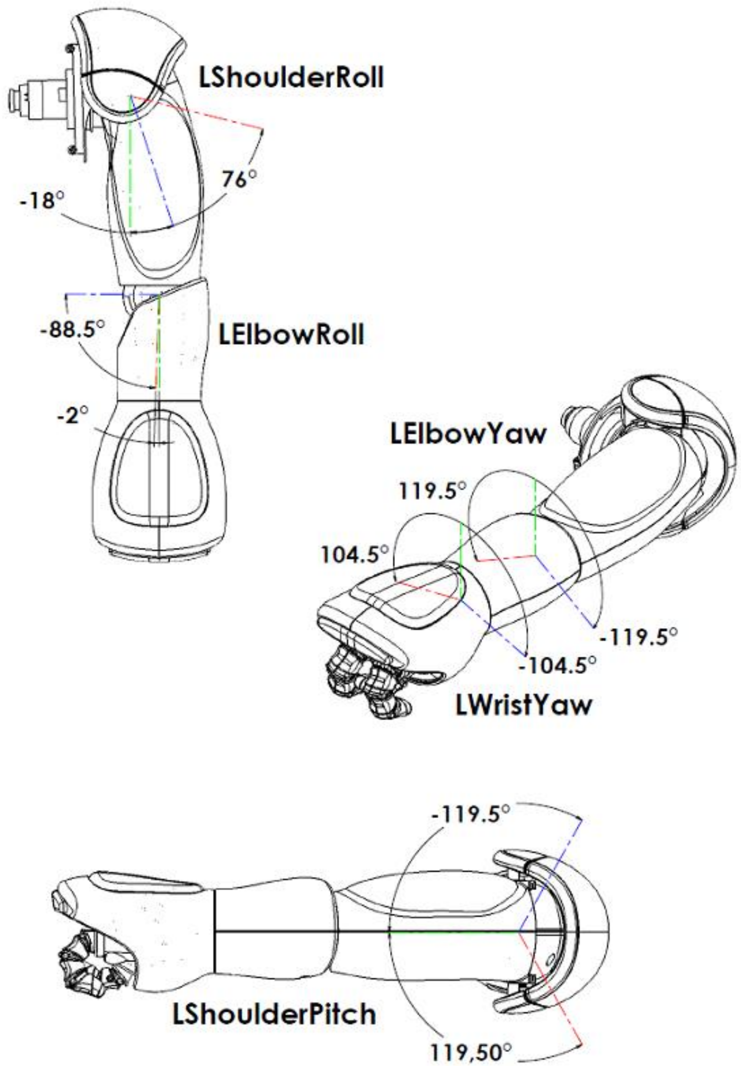


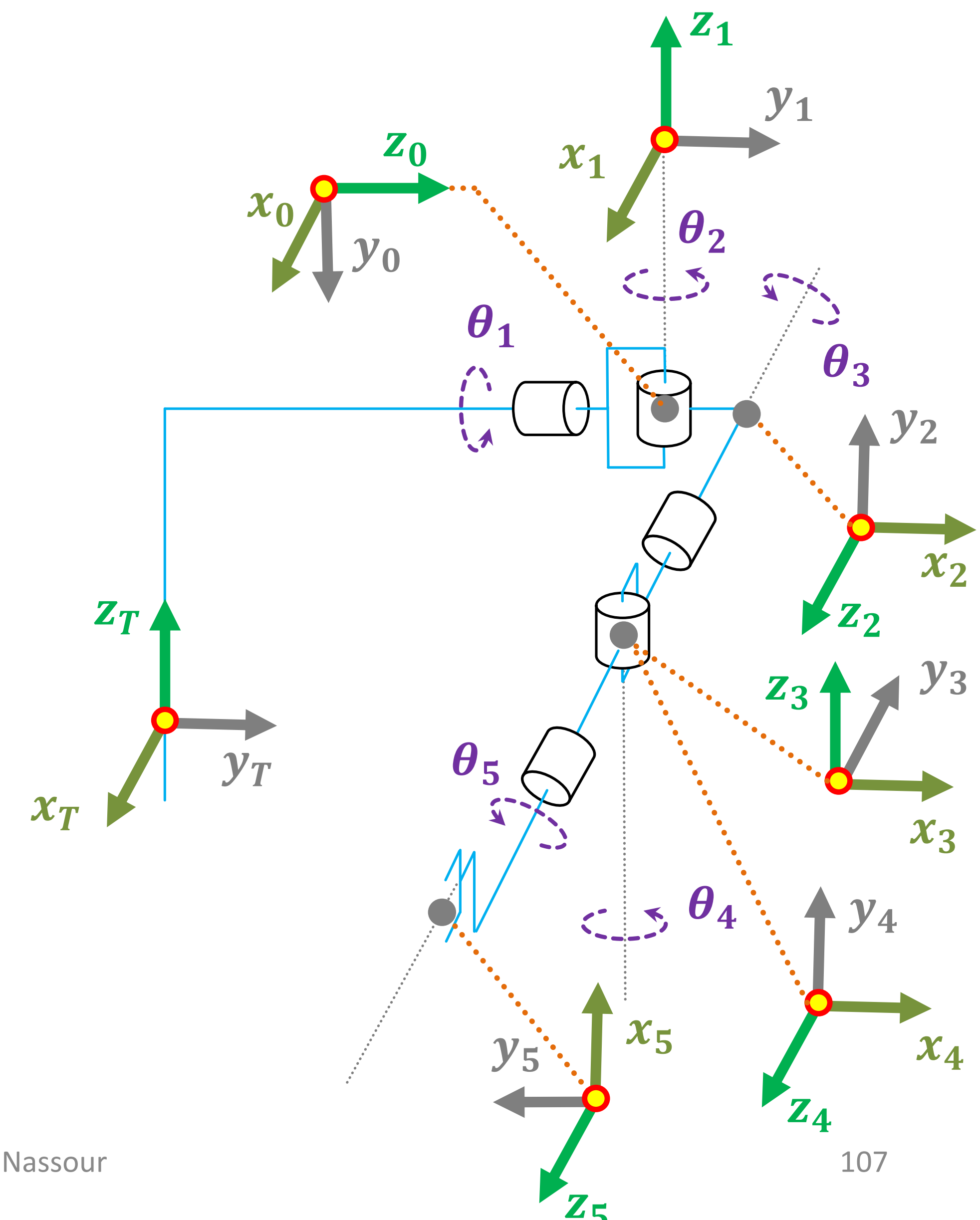
Give two examples of singularities that can occur.

Whenever $\theta_5 = 0$, the manipulator is in a singular configuration because joint 4 and 6 line up. Both joints actions would results the same end-effector motion (one DOF will be lost).

Whenever $\theta_3 = -90$, the manipulator is in a singular configuration. In this situation, the arm is fully extracted. This is classed as a workspace boundary singularity.

NAO Left Arm


LShoulderRoll
76°
-18°
-88.5°
LElbowRoll
-2°
LElbowYaw
119.5°
104.5°
-119.5°
-104.5°
LWristYaw
-119.5°
LShoulderPitch
119,50°


z_1
y_1
x_1
z_0
x_0
y_0
θ_2
θ_1
θ_3
y_2
x_2
z_2
z_T
y_T
x_T
z_3
y_3
x_3
θ_5
θ_4
y_4
x_4
z_4
y_5
x_5
z_5

NAO Right Arm