Winter School “Geometric and Harmonic Analysis”

Program

Monday, February 17th

9:00 – 10:30 Diego Pallara: *An introduction to infinite dimensional analysis I*

Coffee and tea

11:00 – 12:30 Felix Pogorzelski: *On sofic groups, local empirical convergence and spectral approximation I*

Lunch

14:00 – 15:30 Marcel Schmidt: *Spectral theory v.s. metric geometry on Riemannian manifolds I*

Coffee and tea

16:00 – 16:30 Elias Döhrer (Chemnitz): *Differential geometry meets infinite dimensional calculus: A Riemannian metric on the space of embeddings* $\mathbb{S}^1 \to \mathbb{R}^m$

16:40 – 17:10 Elias Zimmermann (Leipzig): *Exponential mixing and spherical equipartition for processes on trees*

17:30 – 18:30 *Q & A with the speakers*

19:00 *Dinner*

Tuesday, February, 18

9:00 – 10:30 Marcel Schmidt: *Spectral theory v.s. metric geometry on Riemannian manifolds II*

Coffee and tea

11:00 – 12:30 Diego Pallara: *An introduction to infinite dimensional analysis II*

Lunch

14:00 – 15:30 Felix Pogorzelski: *On sofic groups, local empirical convergence and spectral approximation II*

Coffee and tea

16:00 – 16:30 Kathrin Völkner (Berlin): *Cheeger Problems on weighted Riemannian Manifolds with Applications in Dynamical Systems*

16:40 – 17:10 Eduardo Alejandro Silva Müller (Münster): *Continuity of asymptotic entropy on wreath products*

18:30 *Dinner*

Wednesday, February, 19

9:00 – 10:30 Felix Pogorzelski: *On sofic groups, local empirical convergence and spectral approximation III*

Coffee and tea

11:00 – 12:30 Marcel Schmidt: *Spectral theory v.s. metric geometry on Riemannian manifolds III*

Lunch

14:00 – 15:30 Diego Pallara: *An introduction to infinite dimensional analysis III*

Coffee and tea

16:00 – 16:30 Maxime Marot (Chemnitz): *Heat Kernel analysis on Alexandrov surfaces*

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Campus map: `https://www.tu-chemnitz.de/tu/lageplan/rhstr.php.en`

Abstracts

- **Diego Pallara** (Lecce)

  An introduction to infinite dimensional analysis

  In this lecture series the basic notions of Malliavin calculus in infinite dimensional separable Banach or Hilbert spaces endowed with a Gaussian measure are presented. This theory can be regarded as an infinite dimensional variant of Sobolev spaces. The Ornstein-Uhlenbeck semigroup and its generator will be discussed, together with the main properties of Sobolev spaces and the space of BV functions.

- **Felix Pogorzelski** (Leipzig)

  On sofic groups, local empirical convergence and spectral approximation

  The notion of local empirical convergence (le-convergence) towards invariant measures on configuration spaces has been defined by Austin for a study on entropy for measure preserving actions of sofic groups. As it turns out, le-convergence is also a useful concept in aspects of spectral convergence for certain discrete random operators. The goal of the lecture series is to explain this connection for random Schrödinger type operators on sofic Cayley graphs. We demonstrate how to prove a probabilistic uniform approximation theorem for the integrated density of states via eigenvalue counting functions on finite-volume analogs. A special focus will be on periodically approximable groups (PA-groups), for which le-convergent approximations always exist.

- **Marcel Schmidt** (Jena)

  Spectral theory v.s. metric geometry on Riemannian manifolds

  This lecture series gives an introduction on the interplay between the metric geometry and the spectral theory of the Laplace-Beltrami operator on a Riemannian manifold. Self-adjoint realizations of the Laplacian are discussed, as well as bounds for the bottom of its spectrum, and some spectral theory on L^∞, with the latter being related to global properties of Brownian motion. The involved methods are so robust that they generalize to large classes of less smooth spaces (e.g. metric measure spaces and discrete graphs).

- **Elias Döhrer** (Chemnitz)

  Differential geometry meets infinite dimensional calculus: A Riemannian metric on the space of embeddings $\mathbb{S}^1 \to \mathbb{R}^m$

  Arnold introduced a new perspective in the field of geometric analysis and geometric mechanics: infinite dimensional differential geometry. Ebin and Marsden expanded on his ideas and developed a more general framework. This gave rise to a new branch in mathematics.

In my talk, I will explain the beginning of their investigations via an example. After that, I am going to introduce the geometry of mapping spaces. This will lead us to the setting of embeddings and the occurring difficulties. Since the field of infinte dimensional geometry is quite broad and still under heavy research, I will restrict myself to "strong Riemannian metrics". My colleagues and I designed a Riemannian metric on the space of embeddings of the circle. Inspired by the tangent-point energy, we were able to incorporate self-repulsion into a metric on the mentioned manifold. I will elaborate on how we were able to overcome the topological obstructions of the manifold and some basic concepts. In the second half of the talk I will explain the result we were able to achieve and give short proofs for some theorems. This will clarify the analytical difficulties of this theory. After that I will finish with elaborating on how our geometrical statements affect the dynamics of the deformations and give an outlook onto future work and other directions open for exploration.

- **Maxime Marot** (Chemnitz)

  Heat Kernel analysis on Alexandrov surfaces

  Our goal is to prove the existence of a heat kernel on an Alexandrov surface with bounded integral curvature. Roughly speaking, these surfaces are the less regular ones such that the Gauß-Bonnet theorem holds and there exist conformal coordinates. After this introduction is done, we will see that, locally, they support a Poincaré inequality and their measure are doubling. Then the theory developed by Sturm applies and yields a jointly continuous heat kernel.

- **Eduardo Alejandro Silva Müller** (Münster)

  Continuity of asymptotic entropy on wreath products

  The asymptotic entropy of a random walk on a countable group is a non-negative number that determines the existence of non-constant bounded harmonic functions on the group. A natural question to ask is whether the asymptotic entropy, seen as a function of the step distribution of the random walk, is continuous. This question has been studied for several classes of groups, some of which exhibit continuity for all probability measures with finite Shannon entropy (e.g. Gromov hyperbolic groups), whereas others exhibit discontinuity (e.g. the group of finitely supported permutations of the integers). In this talk, I will explain recent results of mine on the continuity of asymptotic entropy for non-degenerate probability measures with finite Shannon entropy on wreath products $A \wr \mathbb{Z}^d$, where A is a countable group and $d \geq 3$.

- **Kathrin Völkner** (Berlin)

  Cheeger Problems on weighted Riemannian Manifolds with Applications in Dynamical Systems

  Given a relatively compact domain in a Riemannian manifold, classical Cheeger problems consist of finding a subset which minimizes the ratio between its boundary surface measure and its volume measure among all smooth subsets of the domain in an appropriate sense. These problems have applications in the study of so-called coherent sets in dynamical systems. In this context, one studies Cheeger problems with respect to time-dependent families of Riemannian metrics and volume measures on the manifold. Two types of problems are of interest here: a time-averaged Cheeger problem one the one hand, and a parameter-dependent Cheeger problem with respect to an induced time-fibered Riemannian metric on the other hand. We formulate both problems in the setting of functions of bounded variations and show how the former problem arises as the Gamma limit of the parameter-dependent problem.

- **Elias Zimmermann** (Leipzig)

  Exponential mixing and spherical equipartition for processes on trees

  Let $G = (V, E)$ be a Cayley graph and Λ be a finite set. We say that a stochastic process $(\sigma_v)_{v \in V}$ taking values in Λ satisfies the asymptotic equipartition property (AEP) along a sequence $(F_n)_{n=1}^{\infty}$ of finite sets $F_n \subseteq V$ if the distribution of the processes $(\sigma_v)_{v \in F_n}$ becomes closer and closer to equipartition as n goes to ∞. For stationary and ergodic processes on $\mathbb{Z}$ the AEP is a consequence of the famous Shannon-McMillan-Breiman (SMB) theorem. Initiated by the groundbreaking work of Ornstein and Weiss the SMB theorem has been extensively generalized to graphs with amenable geometry yielding the AEP along suitable sequences of Følner sets such as cubes in $\mathbb{Z}^d$ or balls in Cayley graphs of polynomial volume growth.

  However, for Cayley graphs with a hyperbolic geometry only few results on equipartition exist so far. For instance, Berger and Ye could establish the AEP along balls for automorphism invariant and ergodic processes on regular trees, which arise as the Cayley graphs of free groups or free products of cyclic groups. Moreover, recent results of Nevo and Pogorzelski, which utilize the fact that those groups act amenably on the boundary of the tree, give rise to horospherical and geodesic equipartition theorems. In this talk I will explain how one can build on the latter approach and use a quantitative mixing condition to obtain equipartition along spheres in this setting. Examples of appropriately mixing processes include Gibbs states of Ising models at high temperatures and Potts models with many spin states. The talk is based on joint work with Felix Pogorzelski.