Probabilistic error bounds for the Moving Least Squares (MLS) approximation on scattered data

The MLS approximation is a form of scattered data approximation. Given a function $f\colon \Omega \to \mathbb{R}$ and some center $x \in \Omega \subset \mathbb{R}^d$, one first computes a local least squares approximation on a small region $\Omega_x \subset \Omega$ containing x, by using only the data contained in Ω_x. The MLS approximation of f at x is defined as the value of the local least squares approximation at x.

We show that the error of the MLS approximation at x is essentially bounded by the ratio between the continuous and the discrete L^2 norm on the local domain Ω_x, multiplied by the square root of the related Christoffel function at x. The main difference to error estimates for the standard least squares approximation is that for every center x, the Christoffel function must only be bounded at x and not on all of Ω_x.

Considering randomly drawn nodes, we also discuss the application of random matrix theory and Marcinkiewicz-Zygmund inequalities, in order to obtain probabilistic error bounds.