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CoBarS: Fast reweighted sampling for polygon spaces in any dimension

I will report on a recent project with Jason Cantarella (University of Georgia). We developed a novel algorithm for sampling random configurations of closed n-gons with any prescribed edgelengths $r_1, \dots, r_n \in \mathbb{R}_+$ in any dimension d. We prove that it samples correctly from standard probability measures on these spaces. Formerly known algorithms for this were either restricted to dimensions $d = 2$ or $d = 3$, or they were restricted to equilateral polygons. Moreover, we prove that the new algorithm has runtime complexity $O(n)$, while the best previously known sampler has complexity $O(n^{5/2})$.

The idea behind the process is to first sample unit edge vectors uniformly on the unit sphere. Then we find the so-called *conformal barycenter* of this point cloud with respect to the weights $r_1, \dots, r_n$ by a Riemannian optimization in hyperbolic space. This conformal barycenter helps us to construct a Möbius transformation of the sphere so that the transformed unit edge vectors close up to a closed polygon. The procedure induces some bias in the sampling that can be removed by certain reweighting factors. The central achievement of our works is to compute these reweighting factors in $O(n)$ time.