Adaptive Vertex-centered Finite Volume Methods (Petrov-Galerkin) with Convergence Rates for General Second-Order Linear Elliptic PDE

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A classical finite volume method describes numerically a conservation law of an underlying model problem. It naturally preserves local conservation of the numerical fluxes. Therefore, finite volume methods are well-established in the engineering community (fluid mechanics). In this lecture, we approximate the solution of a general second-order linear elliptic PDE with an adaptive vertex-centered finite volume method (FVM). Note that we can write the vertex-centered FVM with first-order conforming ansatz functions on a primal mesh and piecewise constant test functions on the corresponding dual mesh also in variational form (Petrov-Galerkin method). The adaptive mesh-refinement is driven by the local contributions of a weighted-residual error estimator. We prove that the adaptive algorithm leads to linear convergence with generically optimal algebraic rates for the error estimator and the sum of energy error plus data oscillations. Similar results have been derived for finite element methods and boundary element methods. However, the lack of the classical Galerkin orthogonality for FVM leads to new challenges. For non-symmetric model problem configurations, we additionally have to prove a new L^2-type estimate.

References:

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