TECHNISCHE UNIVERSITÄT CHEMNITZ

Faculty of Mathematics
Inverse Problems

Chemnitz Symposium on Inverse Problems 2016

Conference Guide

September 22 – 23, 2016

Chemnitz, Germany

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List of participants

Imprint

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General information

Goal

Our symposium will bring together experts from the German and international 'Inverse Problems Community' and young scientists. The focus will be on ill-posedness phenomena, regularization theory and practice, and on the analytical, numerical, and stochastic treatment of applied inverse problems in natural sciences, engineering, and finance.

Location

Technische Universität Chemnitz
Straße der Nationen 62 (Böttcher-Bau)
Conference hall 'Altes Heizhaus'
09111 Chemnitz, Germany

Selection of invited speakers

Bangti Jin (London, UK)
Lothar Reichel (Kent, USA)
Frank Werner (Göttingen, Germany)

Scientific board

Bernd Hofmann (Chemnitz, Germany)
Peter Mathé (Berlin, Germany)
Sergei V. Pereverzyev (Linz, Austria)
Masahiro Yamamoto (Tokyo, Japan)

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Timetable

Overview for Thursday, September 22

09.00–09.05	**Opening**
09.05–10.15	**Session 1** L. Reichel, B. Jin
10.15–10.35	**Coffee break**
10.35–11.45	**Session 2** B. Kaltenbacher, V. Michel, F. Weidling
11.45–12.55	**Lunch break**
12.55–14.10	**Session 3** P. Mathé, S. Pereverzyev, J. Flemming
14.10–14.25	**Coffee break**
14.25–16.25	**Session 4** E. Klann, D. Gerth, P. Tkachenko, A. Rahimov, M. Quellmalz
16.40–22.00	**Excursion**

Overview for Friday, September 23

08.30–09.55	**Session 1** F. Werner, M. Yamamoto, A. Rösch
09:55–10.15	**Coffee break**
10.15–11.40	**Session 2** D. Garmatter, A. Wald, P. Gralla, S. Bürger
11.40–12.50	**Lunch break**
12.50–14.00	**Session 3** I. Shestakov, A. Sakat, S. Hubmer
14.00–14.10	**Coffee break**
14.10–14.50	**Session 4** M. Kontak, V. Hutterer

Program for Thursday, September 22

09.00–09.05	**Opening**
09.05–09.40	**Lothar Reichel** (Kent, USA) *Generalized Krylov subspace methods for l_p-l_q minimization*
09.40–10.15	**Bangti Jin** (London, UK) *Sparse recovery by ℓ^0 penalty*
10.15–10.35	**Coffee break**
10.35–11.00	**Barbara Kaltenbacher** (Klagenfurt, Austria) *Regularization by bound constraints and its application to parameter identification in PDEs*
11.00–11.25	**Volker Michel** (Siegen, Germany) *Observing Climate Change | Some Solved and Unsolved Mathematical Problems*
11.25–11.45	**Frederic Weidling** (Göttingen, Germany) *Characterizations of variational source conditions, converse results, and maxisets of spectral regularization methods*
11.45–12.55	**Lunch break**
12.55–13.20	**Peter Mathé** (Berlin, Germany) *Complexity of linear ill-posed problems in Hilbert space I*
13.20–13.45	**Sergei Pereverzyev** (Linz, Austria) *Complexity of linear ill-posed problems in Hilbert space II*
13.45–14.10	**Jens Flemming** (Chemnitz, Germany) *Convergence rates for ℓ^1-regularization without injectivity-type assumptions*
14.10–14.25	**Coffee break**

14.25–14.50	**Esther Klann** (Berlin, Germany) *Topological derivatives for domain functionals with an application to tomography*
14.50–15.15	**Daniel Gerth** (Chemnitz, Germany) *On oversmoothing regularization*
15.15–15.40	**Pavlo Tkachenko** (Linz, Austria) *Nyström type subsampling analyzed as a regularized projection*
15.40–16.05	**Anar Rahimov** (Marseille, France) *Numerical solution to inverse problems for parabolic equation with nonlocal conditions*
16.05–16.25	**Michael Quellmalz** (Chemnitz, Germany) *A generalization of the Funk–Radon transform*
16.40–22.00	Excursion to Villa Esche and conference dinner

Time periods include 5 minutes for discussion.

Program for Friday, September 23

08.30–09.05	**Frank Werner** (Göttingen, Germany) *Support inference in linear statistical inverse problems*
09.05–09.30	**Masahiro Yamamoto** (Tokyo, Japan) *Well-posedness of initial - boundary value problems for time-fractional diffusion equations and inverse problems*
09.30–09.55	**Arnd Rösch** (Essen, Germany) *A semilinear parabolic problem with a directional sparsity functional*
09.55–10.15	**Coffee break**
10.15–10.35	**Dominik Garmatter** (Frankfurt, Germany) *Reduced basis methods for nonlinear ill-posed inverse problems*
10.35–10.55	**Anne Wald** (Saarbrücken, Germany) *Solving nonlinear inverse problems by sequential subspace optimization with an application to terahertz tomography*
10.55–11.15	**Phil Gralla** (Bremen, Germany) *Inverse Problems Incorporating Tolerances*
11.15–11.40	**Steven Bürger** (Chemnitz, Germany) *Discretized Lavrent'ev regularization for the autoconvolution equation*
11.40–12.50	**Lunch break**

12.50–13.15	**Ivan Shestakov** (Oldenburg, Germany) *On an ill-posed mixed problem for parabolic systems*
13.15–13.40	**Abdeljalil Sakat** (Safi, Morocco) *Explicit a posteriori error estimates for recovering boundary data*
13.40–14.00	**Simon Hubmer** (Linz, Austria) *Inverse Problems and MRAI - Mapping the pulse wave velocity*
14.00–14.10	**Break**
14.10–14.30	**Max Kontak** (Siegen, Germany) *A greedy algorithm for the solution of nonlinear inverse problems*
14.30–14.50	**Victoria Hutterer** (Linz, Austria) *The Inverse Problem of Wavefront Reconstruction from Pyramid Sensor Data*

Time periods include 5 minutes for discussion.

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Discretized Lavrent'ev regularization for the autoconvolution equation

Steven Bürger, Peter Mathé

Lavrentiev regularization for the autoconvolution equation was considered by J. Janno in *Lavrentiev regularization of ill-posed problems containing nonlinear near-to-monotone operators with application to autoconvolution equation*, Inverse Problems, 16(2):333–348, 2000. Here this study is extended by considering discretization of the Lavrentiev scheme by splines. It is shown how to maintain the known convergence rate by an appropriate choice of spline spaces and a proper choice of the discretization level. For piece-wise constant splines the discretized equation allows for an explicit solver, in contrast to using higher order splines. This is used to design a fast implementation by means of post-smoothing, which provides results, which are indistinguishable from results obtained by direct discretization using cubic splines.

Convergence rates for ℓ^1-regularization without injectivity-type assumptions

Jens Flemming

To prove convergence rates for ℓ^1-regularization in infinite-dimensional spaces usually injectivity-type assumptions on the linear operator are required. Especially finite basis injectivity or the restricted isometry property are used. In the talk we present a technique to avoid such injectivity-type assumptions while obtaining the same convergence rates as with injective operators.

Reduced basis methods for nonlinear ill-posed inverse problems

Dominik Garmatter, Bastian Harrach, Bernard Haasdonk

The numerical solution of nonlinear inverse problems such as the identification of a parameter in a partial differential equation (PDE) from a noisy solution of the PDE via iterative regularization methods, e.g. the Landweber method or Newton-type methods, usually requires numerous amounts of forward solutions of the respective PDE. One way to speed up the solution process therefore is to reduce the computational time of the forward solution, e.g. via the reduced basis method.

The reduced basis method is a model order reduction technique which constructs a low-dimensional subspace of the solution space. Galerkin projection onto that space allows for an approximative solution. An efficient offline/online decomposition enables the rapid computation of the approximative solution for many different parameters.

This talk will discuss various ways of combining the reduced basis method with existing solution algorithms for nonlinear ill-posed inverse problems, where the main topic of the talk is the development of the new Redued Basis Landweber (RBL) method [1]. The general idea of the method is to adaptively construct a small, problem-oriented reduced basis space instead of constructing a global reduced basis space like it is normally the case in reduced basis methods. This will be done in an iterative procedure: the inverse problem will be solved up to a certain accuracy with a Landweber method that is projected onto the current reduced basis space. The resulting parameter then is utilized to enrich the reduced basis space and therefore fit it to the given problem. This iteration is performed until an iterate is accepted as the solution of the inverse problem. Numerical results will show a significant speed-up and the possibility to reconstruct very high-dimensional parameters.

References

[1] D. Garmatter, B. Haasdonk, and B. Harrach. A reduced basis Landweber method for nonlinear inverse problems. *Inverse Problems*, 32(3):035001, 2016.

On oversmoothing regularization

Daniel Gerth, Bernd Hofmann

In recent years, sparsity promoting regularization was one of the prime topics in the Inverse Problems community. Assuming sparsity of the solution, it was shown that this regularization approach renders the formerly ill-posed problem essentially well-posed. It may, however, be of interest to find a sparse approximation to a non-sparse true solution of the Inverse Problem, for example to extract certain features of the solution. This is a special case of oversmoothing regularization, where it is known a-priori that the sought-after solution does not fulfill the smoothness properties implied by the regularization method. In this case, not much is known about its convergence properties. This talk will follow and intertwine two main threads. On one hand, we discuss convergence properties of general oversmoothing regularization methods. On the other hand we demonstrate the results and extend them to convergence rates for the case of Tikhonov regularization with ℓ^1-penalty for a diagonal operator where the solution is assumed to be in $\ell^2 \backslash \ell^1$.

Inverse Problems Incorporating Tolerances

Phil Gralla, Iwona Piotrowska-Kurczewski, Peter Maaß

Tikhonov functionals for solving inverse problems consist of a discrepancy and a penalty term. The discrepancy term evaluates the deviation of the operator evaluation from the measured data. We propose to adjust the discrepancy term by incorporating a tolerance measure, which neglects deviations from the data within a prescribed tolerance. This is motivated e.g. by applications where multiple measurements of the same object are available. Thus providing a confidence area for the true data. In this talk we provide first analytic results concerning existence, uniqueness and convergence properties of minimizers for this type of Tikhonov functionals incorporating tolerance.

Inverse Problems and MRAI - Mapping the Pulse Wave Velocity

Simon Hubmer, Andreas Neubauer,
Ronny Ramlau, Henning Voss

Magnetic Resonance Advection Imaging (MRAI) is a recently developed method for mapping the pulsatory signal component of dynamic EPI data of the brain, such as acquired in functional and resting state functional MRI experiments. Its underlying model is based on an advection equation. It has been shown that MRAI depicts the location of major arteries as well as some venous structure. In addition, colour direction maps allow for visualization of the orientation direction of blood vessels.

It has been suggested that MRAI may potentially serve as a biomarker for the health of the cerebrovascular system. The reason for this is that MRAI is designed to reflect the spatiotemporal properties of travelling waves, and pulse wave velocities (PWV) are a main indicator for the physical properties of blood vessels.

First results in MRAI were achieved using a multiple regression approach for the underlying advection equation. In this talk, we present a different approach which is based on a classical inverse problem of parameter estimation type. The resulting method for estimating the PWV in the brain from functional MRI data yields improved results, as will be demonstrated numerically using a model problem and a real-world MRI data set.

The Inverse Problem of Wavefront Reconstruction from Pyramid Sensor Data

Victoria Hutterer, Ronny Ramlau

Objects on the sky suffer from distortions and therefore appear blurred when observed by a ground based telescope due to diffraction of light and turbulences in the atmosphere. Adaptive Optics (AO) is a technology that compensates for the rapidly changing optical perturbations arising during the imaging process and physically corrects these turbulences via deformable mirrors in real-time. Measurements of the starlight wavefront deformations are obtained by wavefront sensors which generally provide intensity data related in a non-linear way to the wavefront of the incoming light. Restoration of the unknown wavefront from given sensor measurements is an inverse problem. For the new generation of Extremly Large Telescopes having mirror diameters of around 40m, the computational load makes existing control algorithms such as matrix vector multiplication unfeasible.

In this talk we investigate fast wavefront reconstruction methods for pyramid sensor data. A pyramid wavefront sensor is based on an oscillating pyramidic optical component that splits the light in four beams providing information on the derivatives of the aberrated wavefront. We will consider both, non-modulated and modulated pyramid-type wavefront sensor data.

Sparse recovery by ℓ^0 penalty

Bangti Jin

Sparsity is an important tool in many signal and image processing tasks. However, this is generally achieved by the convex approach with an ℓ^1 penalty. The golden standard, the ℓ^0 penalty, is widely dismissed as intractable, due to its combinatorial nature. In this talk, I will describe some recent progresses in theoretical justifications and algorithmic developments of regularization by the ℓ^0 penalty.

Regularization by bound constraints and its application to parameter identification in PDEs

Barbara Kaltenbacher

In this talk we present a method for the regularized solution of nonlinear inverse problems, based on Ivanov regularization (also called method of quasi solutions). In this talk we first of all compare the three classical paradigms Tikhonov, Ivanov and Morozov regularization, shortly dwell on the convergence analysis of the latter two and then discuss applicability to some parameter identification problems in elliptic PDEs. This is joint work with Christian Clason and Andrej Klassen, University of Duisburg-Essen.

Topological derivatives for domain functionals with an application to tomography

Esther Klann

We study the topological sensitivity of the piecewise constant Mumford–Shah type functional for linear ill-posed problems. We consider a linear operator $K : X \to Y$ and noisy data g^δ approximating $g = Kf$ where f is the function we are interested in. We assume $f : D \to \mathbb{R}$, $D \subset \mathbb{R}^2$ and

$$f = \sum_{i=1}^{m} c_i \chi_{\Omega_i} \quad \text{with} \quad c_i \in \mathbb{R},\ \Omega_i \subset \mathbb{R}^2 \quad \text{and} \quad \chi_D = \sum_{i=1}^{m} \chi_{\Omega_i},$$

i.e., f is a piecewise constant function and the sets Ω_i are a partition of the domain of definition D. We study the topological sensitivity of the Mumford–Shah-type functional

$$J(\vec{c}, \vec{\Omega}) := \|Kf - g^\delta\|_{L_2}^2 + \alpha \sum_{i=1}^{m} |\partial \Omega_i|,$$

i.e., its reaction to a change in topology such as inserting or removing a set Ω_j. The topological derivative indicates if such a change in the topology will decrease the value of the Mumford-Shah-type functional, thus it can be used to find a minimizer of the functional and a solution to the reconstruction problem.

We use the topological derivative in an application from tomographic imaging (with the Radon transform as operator) to find inclusions in an object.

References

[1] R. Ramlau and W. Ring. A Mumford-Shah approach for contour tomography. *Journal of Computational Physics*, Volume 221, Issue 2, 10 February 2007, Pages 539-557.

[2] M. Hintermüller and A. Laurain. Multiphase Image Segmentation and Modulation Reconvery based on Shape and Topological Sensitivity. *Journal of Mathematical Imaging and Vision*, September 2009, Volume 35, Issue 1, pp 1-22

A Greedy Algorithm for the Solution of Nonlinear Inverse Problems

Max Kontak

Both linear and nonlinear inverse problems play a significant role in applications, such as the geosciences. In recent years, the Geomathematics Group at the University of Siegen has developed a greedy algorithm for the solution of linear inverse problems on spherical geometries like the Earth, called the *Regularized Functional Matching Pursuit (RFMP)*. The approximate solution generated by the algorithm is sparse and can consist of both global and highly localized basis functions, such as spherical harmonics and reproducing kernels, respectively. Consequently, very different kinds of structures in the solution can be approximated by appropriate basis functions.

In this talk, we extend the concept of the RFMP to nonlinear inverse problems and present numerical results, where we applied the algorithm to the nonlinear inverse problem of gravimetry, i. e., the inversion of the nonlinear integral operator

$$\sigma \mapsto \int_{\mathbb{S}^2} \int_0^{\sigma(\xi)} \frac{1}{|\cdot - r\xi|} \, r^2 \, \mathrm{d}r \, \mathrm{d}\omega(\xi),$$

where $\sigma \colon \mathbb{S}^2 \to (0, \infty)$. We will also compare this newly developed algorithm to other well-known algorithms for nonlinear inverse problems, such as the Landweber iteration and the Levenberg-Marquardt method.

Complexity of linear ill-posed problems in Hilbert space I

Peter Mathé

We shall shed light on a rigorous complexity analysis of ill-posed problems for two noise models, bounded deterministic and stochastic (Gaussian white) noise. In a seminal paper by A. G. Werschulz, cf. cite[1],[2] it was asserted that linear ill-posed problems in Hilbert space have infinite complexity. As we shall discuss, this limitation can be overcome by imposing solution smoothness. In this talk we shall establish (tight) lower and upper bounds on the information complexity for bounded deterministic noise.

This is the first part of the joint study with Sergei V. Pereverzev (RICAM, Linz).

References

[1] A. G. Werschulz: *An information-based approach to ill-posed problems*, J. Complexity **3** (1987), no. 3, 270–301.

[2] A. G. Werschulz: *What is the complexity of ill-posed problems?*, Numerical Functional Analysis and Optimization **9** (1987), no. 9-10, 945–967.

Observing Climate Change — Some Solved and Unsolved Mathematical Problems

Volker Michel

One striking effect of climate change is the melting of glaciers, for example in Greenland. As a support of political decisions, an accurate observation of the decay of the ice sheets is evidently necessary. For this purpose, a satellite mission called GRACE (Gravity Recovery And Climate Experiment) was launched in 2002 and is still operating. It has provided us with monthly models of the gravitational field of the Earth. Since the gravitational field is a consequence of the mass distribution of the Earth, it is necessary to solve an inverse problem represented by a Fredholm integral equation of the first kind (i.e. Newton's Law of Gravitation).
Though this problem is already severely ill-posed (non-unique solution and exponential sequence of singular values), it is not the only mathematical problem which has to be solved, before a reasonable estimate for the melted ice sheet can be made. In particular, the following problems occur:

- Due to an aliasing effect, the GRACE models are contaminated with noise which occurs as stripes in the plots of the gravitational field. Therefore, an appropriate smoothing/denoising has to be applied.

- The ill-posed inverse problem of gravity inversion for masses has to be regularized and the solution has to be represented in an appropriate basis which allows a regional modelling on a sphere (i.e. the Earth's surface).

- Due to the elastic properties of the Earth, a temporally varying ice load on the Earth's crust causes deformations in the upper parts of the Earth. Theses deformations themselves change the gravitational field. This effect has to be compensated for, since otherwise the estimated mass loss is not accurate enough. From the mathematical point of view, this effect causes a coupling of the aforementioned inverse problem with a partial differential equation.

In this talk, an overview and a brief insight is given with respect to the mathematical problems that occur in this context. Selected tools for solving them are explained and open problems are discussed.
This is a joint project with J Frohne, J Kusche, R Rietbroek, F-T Suttmeier, and R Telschow.

Complexity of linear ill-posed problems in Hilbert space I

Peter Mathé, Sergei V. Pereverzyev

This is the second part of the joint study with Peter Mathe (WIAS-Berlin). It is common belief that stochastic Gaussian white noise makes ill-posed problems more complex than problems under deterministic bounded noise. In this study we shed light on a rigorous complexity analysis of ill-posed problems providing (tight) lower and upper bounds for both noise models. It will be shown that in contrast to the deterministic case statistical ill-posed problems have finite complexity at every prescribed error level.

A generalization of the Funk–Radon transform

Michael Quellmalz

The FunkRadon transform (a.k.a. spherical Radon transform) assigns to a function on the two-sphere its mean values along all great circles. We consider the following generalization: we replace the great circles by the small circles being the intersection of the sphere with planes containing a common point ζ inside the sphere. If ζ is the origin, this is just the FunkRadon transform.

We describe a purely geometric mapping from the sphere to itself that enables us to represent the generalized Radon transform in terms of the FunkRadon transform. This representation is utilized to characterize the nullspace and range as well as to show an inversion formula of the generalized Radon transform.

References

[1] Y. Salman. An inversion formula for the spherical transform in S^2 for a special family of circles of integration. *Anal. Math. Phys.*, 6(1): 43 – 58, 2016.

[2] M. Quellmalz. A generalization of the Funk–Radon transform to circles passing through a fixed point. Preprint 2015-17, Faculty of Mathematics, TU Chemnitz, 2015. Available at `https://tu-chemnitz.de/~qmi/paper/sphericalTransform.pdf`

Numerical solution to inverse problems for parabolic equation with nonlocal conditions

Anar B. Rahimov

The attention of researchers to nonlocal and coefficient-inverse problems has increased in the recent years. Nonlocal form of initial and boundary conditions is caused by practical impossibility to make measurements of the state of an object (or process) in its separate points or instantly in time. As a rule, this information reflects the state of a process in some neighborhood of the measurement point or on the time interval of the measurement, and sometimes it specifies only the mean value of the state over the entire object or over the entire time interval of its operation. Such problems arise in the investigation of phenomena taking place in plasma, heat propagation and liquid transport processes in capillary-porous media, in the mathematical modeling of the technological process of external gettering in the cleanup of silicon wafers from impurities, and in problems of mathematical biology, demography etc.

We consider two inverse problems for a parabolic equation under nonlocal, final, and boundary conditions. The specific character of the considered classes of inverse problems is that the identifiable coefficient belong to the right-hand side, and they depend only on the time or only on the space. This specific character allows reducing the problem to specially built Cauchy problems with respect to a system of ordinary differential equations.

A numerical method is proposed to solve the inverse source problems, which is based on the use of the method of lines. The initial problems are reduced to a system of ordinary differential equations with unknown parameters. To solve this system we propose an approach based on the sweep method type [1,2]. Thus, we do not use any iterative procedures in the approach proposed in the work. All the necessary computational schemes, formulae, and results of the carried out numerical experiments will be given in the report. The obtained results show the efficiency of the proposed approach.

References

[1] Aida-zade K.R. and Rahimov A.B. 2014 An approach to numerical solution of some inverse problems for parabolic equations, *Inverse Problems in Science and Engineering* Vol. 22, No 1, pp. 96–111.

[2] Aida-zade K.R. and Rahimov A.B. 2015 Solution to classes of inverse coefficient problems and problems with nonlocal conditions for parabolic equations, *Differential Equations*, Vol. 51, No 1, pp. 83–93.

Generalized Krylov subspace methods for l_p-l_q minimization

Lothar Reichel

This talk presents new efficient approaches for the solution of ℓ_p-ℓ_q minimization problems based on the application of successive orthogonal projections onto generalized Krylov subspaces of increasing dimension. The subspaces are generated according to the iteratively reweighted least-squares strategy for the approximation of ℓ_p/ℓ_q-norms by weighted ℓ_2-norms. Computed image restoration examples illustrate the performance of the methods discussed. The combination of a fairly low iteration count and a modest storage requirement makes the proposed methods attractive. This talk presents joint work with G.-X. Huang, A. Lanza, S. Morigi, and F. Sgallari.

A semilinear parabolic problem with a directional sparsity functional

Arnd Rösch

In this talk we study the minimization of a least squares functional connected with a semilinear parabolic equation. We investigate a Tikhonov regularization with an L^2-term and a term forcing directional sparsity. A first challenge is to show existence of a minimizer of the regularized problem. Moreover, we will discuss aspects like optimality conditions, structure and regularity of the minimizer. Another important aspect is the discretization of the problem preserving the directional sparsity property.

This is a joint work with Eduardo Casas (Santander) and Mariano Mateos (Gijon).

EXPLICIT A POSTERIORI ERROR ESTIMATES FOR RECOVERING BOUNDARY DATA

B. Achchab, A. Sakat and A. Souissi

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ABSTRACT

We consider a data completion method for the Cauchy problem of Laplace equation. For an approximate solution given by any one of algorithms of the data completion methods, we give the moment of the error between the true solution and the approximate one. This error indicator is used to improve the approximate solution, to give a knowledge of stability and can be used as a stopping criterion.

1. MAIN FACTS

Let Ω be a Lipschitz bounded domain in $\mathbb{R}^2$. ν is the normal unit to the boundary $\partial\Omega$, oriented outward. We assume that $\partial\Omega$ is partitioned into two open connected portions Γ_C and Γ_I such that $\partial\Omega = \overline{\Gamma}_I \cup \overline{\Gamma}_C$. Each of them is of a non-vanishing measure. The data completion problem for the Laplace equation is given by :
Find (φ, ψ) on Γ_I such that there exists a temperature field u satisfying

$$\begin{cases} \Delta u = 0 \ \text{ in } \Omega, \\ u = f \ \text{ on } \Gamma_C, \\ \dfrac{\partial u}{\partial \nu} = \phi \ \text{ on } \Gamma_C, \end{cases} \tag{1}$$

with $\varphi = u$ and $\psi = \dfrac{\partial u}{\partial \nu}$ on Γ_I.

Let (u_h, w_h) be an approximation of $(u, \dfrac{\partial u}{\partial \nu})$ on Γ_I, by applying any one of the data completion methods see for example [1, 4], we define two error functions on Γ_I, $e_1 = u|_{\Gamma_I} - u_h$ and $e_2 = \dfrac{\partial u}{\partial \nu}|_{\Gamma_I} - w_h$.

In this work, we propose a new procedure, based on the moment technique [2], to explicitly approximate the error e_1 and the error e_2. More precisely, let $\{v_j, j \in \mathbb{N}\}$ be a sequence of orthonormal functions such that $\Delta v_j = 0, \ \forall j \in$ and $\overline{Span\{v_j|_{\Gamma_i}\}_{j=0}^{\infty}} = L^2(\Gamma_I)$. Assume (1) has a solution u such that $u|_{\Gamma_I} \in L^2(\Gamma_I)$. Let m_j^k, $j \in \mathbb{N}$, be the moments of e_k, $k = 1, 2$ defined by $m_j^k = \langle e_k, v_j \rangle = \int_{\Gamma_I} e_k v_j ds$, where $\langle ., . \rangle$ is the inner product of $L^2(\Gamma_I)$. By means of the given data and v_j, $j \in \mathbb{N}$, we give m_j^k $k = 1, 2$. The obtained approximation of e_1 and e_2 permits to build an a posteriori error estimation of residual type [3] in order to improve the given approximate solution and to have a stopping criterion. We give some convergence and stability results and we introduce an algorithm for solving the two moment problems. To illustrate the proposed approach, we give some numerical experiments.

2. ACKNOWLEDGMENT

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3. REFERENCES

[1] S. Andrieux, T. N. Baranger and A. Ben Abda,*Solving Cauchy problems by minimizing an energy-like functional*, Inverse Problems, vol.22, no. 1, (2006) 115-133.

[2] D. D. Ang, R. Gorenflo, L. K. Vy and D. D. Trong, *Moment Theory and Some Inverse Problems in Potential Theory and Heat Conduction*, Lecture Notes in Mathematics 1792, Springer-Verlag Berlin Heidelberg 2002.

[3] R. Becker and R. Rannacher, *An optimal control approach to a posteriori error estimation in finite element methods*, Acta Numerica (2001) 10, 1-102.

[4] A. Chakib and A. Nachaoui, *Convergence analysis for finite element approximation to an inverse Cauchy problem*, Inverse Problems, vol. 22, no. 4(2006) 1191-1206.

On an ill-posed mixed problem for parabolic systems

Ivan Shestakov

Let $\mathcal{X}$ be a bounded domain in $\mathbb{R}^n$ with a smooth boundary $\partial\mathcal{X}$ and $\mathcal{S}$ be an open subset of $\partial\mathcal{X}$. We consider an ill-posed mixed problem for parabolic systems in a cylinder $\partial\mathcal{X} \times (0, T)$ with initial data on the bottom $\mathcal{X} \times \{0\}$ and Cauchy data on the strip $S \times (0, T)$ on the lateral surface. We prove the unique solvability and derive solvability conditions for this problem. Moreover we construct an explicit Carleman formula for solutions of this problem for the heat equation using heat polynomials.

Nyström type subsampling analyzed as a regularized projection

Pavlo Tkachenko

In the statistical learning theory the Nyström type subsampling methods are considered as tools for dealing with big data. In this talk we consider Nyström subsampling as a special form of the projected Lavrentiev regularization, and study it using the approaches developed in the regularization theory. As a result, we prove that the same capacity independent learning rates that are guaranteed for standard algorithms running with quadratic computational complexity can be obtained with subquadratic complexity by the Nyström subsampling approach, provided that the subsampling size is chosen properly. We propose a priori rule for choosing the subsampling size and a posteriori strategy for dealing with uncertainty in the choice of it. The theoretical results are illustrated by numerical experiments.

The work has been performed together with Dr. Galyna Kriukova (RICAM) and Dr. Sergei Pereverzyev JR. (University of Innsbruck).

Solving nonlinear inverse problems by sequential subspace optimization with an application to terahertz tomography

Anne Wald, Thomas Schuster

Sequential Subspace Optimization (SESOP) is a method, which has so far been developed for the solution of linear inverse problems in Hilbert and Banach spaces. The key idea is to use not just one search direction (as in Landweber-type iterations), but multiple search directions in an effort to reduce the computation time. In this talk we want to introduce an adaption of this method for solving nonlinear inverse problems. To this end, we iteratively project the initial value onto stripes whose width is determined by the search direction, the nonlinearity of the operator and the noise level. We discuss convergence and regularization properties of our method and present a fast algorithm that uses two search directions. The talk is concluded with numerical experiments concerning the application of SESOP in terahertz tomography.

Characterizations of variational source conditions, converse results, and maxisets of spectral regularization methods

Frederic Weidling, Thorsten Hohage

We describe a general strategy for the verification of variational source condition by formulating two sufficient criteria describing the smoothness of the solution and the degree of ill-posedness of the forward operator in terms of a family of subspaces. For linear deterministic inverse problems we show that variational source conditions are necessary and sufficient for convergence rates slower than the square root of the noise level. A similar result is shown for linear inverse problems with white noise. If the forward operator can be written in terms of the functional calculus of a Laplace-Beltrami operator, variational source conditions can be characterized by Besov spaces. This is discussed for a number of prominent inverse problems.

Support inference in linear statistical inverse problems

Frank Werner, Axel Munk, Katharina Proksch

Let $\mathcal{X}$ and $\mathcal{Y}$ be separable Hilbert-Spaces, $T : \mathcal{X} \to \mathcal{Y}$ a bounded linear operator and $f \in \mathcal{X}$. Let $\{\psi_j\}_{j\in\mathbb{N}}$ be a complete orthonormal system in $\mathcal{Y}$ and suppose we observe Tf according to the following inverse regression model

$$Y_j = \langle Tf, \psi_i \rangle_{\mathcal{Y}} + \xi_j, \qquad j = 1, \ldots, N. \tag{0.1}$$

Here $\xi_j,\ j = 1, \ldots, N$, are independent, centered and possibly not identically distributed measurement errors, i.e. this model includes white noise as well as e.g. Poissonian observations. Let further $\mathcal{U} = \{\varphi_i\}_{i\in\mathbb{N}} \subset \mathcal{X}$ be a dictionary.

This talk is concerned with inference on the active coefficients of f, i.e. identifying those $i \in \mathbb{N}$ for which $|\langle f, \varphi_i \rangle_{\mathcal{X}}| > 0$. Under suitable assumptions on $\mathcal{U}$ we propose a statistical multiscale method which allows for uniform confidence statements on the recovered support, this is asymptotic guarantees that with prescribed probability no inactive coefficients and all sufficiently strong coefficients have been identified.

Of special interest in (0.1) are convolution operators T between function spaces $\mathcal{X}$ and $\mathcal{Y}$. In this case, we are able to present even sharper results. As an application in nanobiophotonics we consider the detection of positions of markers in a fluorescent sample. The performance of our method is demonstrated by means of a simulation study and real data samples.

Well-posedness of initial - boundary value problems for time-fractional diffusion equations and inverse problems

Masahiro Yamamoto

We discuss initial value - boundary problems for diffusion equations where the fractional orders α of time derivatives are between 0 and 1 and the elliptic part is not symmetric and dependent also on time:

$$\partial_t^\alpha (u(x,t) - a(x)) + A(t)u(x,t) = F(x,t), \quad x \in \Omega,\ 0 < t \le T,$$

$$u(\cdot, t) \in H_0^1(\Omega), \quad 0 < t < T$$

such that $u - a$ is requested in certain Sobolev spaces $\subset H^\alpha(0,T; H^{-1}(\Omega))$. Here

$$(-A(t)u)(x,t) = \sum_{ij=1}^{n} \partial_i (a_{ij}(x,t)\partial_j u(x,t)) + \sum_{j=1}^{n} b_j(x,t)\partial_j u(x,t) + c(x,t)u(x,t), \quad x \in \Omega,\ t > 0,$$

where $a_{ij} = a_{ji} \in C^1([0,T]; C(\overline{\Omega}))$, $1 \le i,j \le n$, satisfy the uniform ellipticity on $\overline{\Omega} \times [0,T]$, and $b_j, c \in L^\infty(\Omega \times (0,\infty))$, $1 \le j \le n$.

Then we define weak solutions by the Caputo time derivatives in Sobolev spaces and prove the unique existence of weak solutions with a priori estimate of the solutions by initial values and right-hand sides of the equations. Moreover we establish the continuity of the solution near $t = 0$. The proof is based on the Galerkin method and the energy estimates. Moreover we apply these results to some inverse problems.

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