Johann Radon Institute for Computational
and Applied Mathematics (RICAM)
Austrian Academy of Sciences (ÖAW)
–
Chemnitz University of Technology
Department of Mathematics
Analysis – Inverse Problems

Chemnitz–RICAM–Symposium on Inverse Problems

Linz, Austria

July 14-15, 2009

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Conference Program

Important Facts

Goal:	Our symposium will bring together experts from the Austrian, German and international 'Inverse Problems Community' and young scientists.The focus will be on ill-posedness phenomena, regularization theory and practice and on the analytical, numerical and stochastic treatment of applied inverse problems from natural sciences, engineering and finance.
Location:	Johann Radon Institute (RICAM) Johannes Kepler University Linz Altenberger Str. 69 (Hochschulfondsgebäude) Room HF9901 4040 Linz, Austria
Dates:	July 14 and July 15, 2009
Scientific Board:	Heinz W. Engl (Vienna and Linz, Austria) Bernd Hofmann (Chemnitz, Germany) Peter Mathé (Berlin, Germany) Sergei V. Pereverzyev (Linz, Austria) Ulrich Tautenhahn (Zittau, Germany)
Organizing Committee:	Bernd Hofmann Magdalena Fuchs Marcus Meyer
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Short program and overview

Day 1: Tuesday, July 14, 2009		Day 2: Wednesday, July 15, 2009	
09.00 - 09.15	**Opening** Heinz W. Engl/Bernd Hofmann	09.00 - 10.30	**Session 5** Hans-Jürgen Reinhardt Ulrich Tautenhahn Antonio Leitão
09.15 - 10.30	**Session 1** Victor Isakov Martin Burger		
10.30 - 11.00	**Break**	10.30 - 11.00	**Break**
11.00 - 12.00	**Session 2** William Rundell Jin Cheng	11.00 - 12.00	**Session 6** Hanna K. Pikkarainen Jaan Janno
12.00 - 13.30	**Lunch Break**	12.00 - 13.30	**Lunch Break**
13.30 - 15.00	**Session 3** Sergei V. Pereverzyev Peter Mathé Masahiro Yamamoto	13.30 - 15.00	**Session 7** Melina Freitag Volker Michel Mourad Sini
15.00 - 15.30	**Break**	15.00 - 15.30	**Break**
15.30 - 17.20	**Session 4** Ronny Ramlau Torsten Hein Philipp Kügler Kristian Bredies	15.30 - 16.30	**Session 8** Uno Hämarik Radu I. Boţ Marcus Grasmair
		16.30 - 16.50	**Break**
		16.50 - 17.35	**Session 9** Marcus Meyer Thomas Bonesky Jens Geissler
		17.35 - 17.45	**Closing and Conclusions**
		18.00	**Conference Dinner** in Restaurant 'Keplers'

Day 1: Tuesday, July 14, 2009

09.00 - 09.15	**Opening** Inaugural address by Heinz W. Engl and Bernd Hofmann
09.15 - 10.30	**Session 1, Chair: Heinz W. Engl (Vienna/Linz, Austria)**
09.15 - 10.00	**Victor Isakov** (Wichita, USA): *"Increasing Stability in Inverse Problems"*
10.00 - 10.30	**Martin Burger** (Münster, Germany): *"Variational Models with Non-Gaussian Noise"*
10.30 - 11.00	**Break**
11.00 - 12.00	**Session 2, Chair: Bernd Hofmann (Chemnitz, Germany)**
11.00 - 11.30	**William Rundell** (Texas, USA): *"Some 'Probe-Methods' for the Reconstruction of Inverse Inclusion Problems"*
11.30 - 12.00	**Jin Cheng** (Shanghai, China): *"The Mathematical Model for the Diffusion Process and Related Inverse Problems"*
12.00 - 13.30	**Lunch Break**
13.30 - 15.00	**Session 3, Chair: Victor Isakov (Wichita, USA)**
13.30 - 14.00	**Sergei V. Pereverzyev** (Linz, Austria): *"A Carleman Estimate and the Balancing Principle in the Quasi-Reversibility Method for Solving the Cauchy Problem for Laplace Equation"*
14.00 - 14.30	**Peter Mathé** (Berlin, Germany): *"On some Minimization-Based Heuristic Parameter Choice in Inverse Problems"*
14.30 - 15.00	**Masahiro Yamamoto** (Tokyo, Japan): *"Inverse Problems for Diffusion Equations of Fractional Orders"*

15.00 - 15.30	**Break**
15.30 - 17.20	**Session 4, Chair: Martin Burger (Münster, Germany)**
15.30 - 16.00	**Ronny Ramlau** (Linz, Austria): *"Tikhonov Regularization with Sparsity Constraints - Regularization Properties, Convergence Rates and Optimization"*
16.00 - 16.30	**Torsten Hein** (Chemnitz, Germany): *"Iterative Regularization in Banach Spaces"*
16.30 - 17.00	**Philipp Kügler** (Linz, Austria): *"An Extended State Approach to Online Parameter Identification in a Class of Infinite Dimensional Dynamical Systems"*
17.00 - 17.20	**Kristian Bredies** (Graz, Austria): *"Inverse Problems with Non-Convex Separable Constraints"*

Day 2: Wednesday, July 15, 2009

09.00 - 10.30	**Session 5, Chair: William Rundell (Texas, USA)**
09.00 - 09.30	**Hans-Jürgen Reinhardt** (Siegen, Germany): *"Multidimensional Inverse Heat Conduction Calculations"*
09.30 - 10.00	**Ulrich Tautenhahn** (Zittau, Germany): *"Conditional Stability Estimates and Regularization with Applications to Cauchy Problems for the Helmholtz Equation"*
10.00 - 10.30	**Antonio Leitão** (Florianopolis, Brazil): *"On Multiple Level Set Type Regularization Methods"*
10.30 - 11.00	**Break**
11.00 - 12.00	**Session 6, Chair: Peter Mathé (Berlin, Germany)**
11.00 - 11.30	**Hanna K. Pikkarainen** (Linz, Austria): *"A Bayesian Model for Root Computation"*
11.30 - 12.00	**Jaan Janno** (Tallinn, Estonia): *"New Uniqueness Results for Parabolic Integro-Differential Identification Problems"*
12.00 - 13.30	**Lunch Break**
13.30 - 15.00	**Session 7, Chair: Arnd Rösch (Duisburg, Germany)**
13.30 - 14.00	**Melina Freitag** (Bath, UK): *"Data Assimilation in Numerical Weather Prediction: 4D-Var and Links to Other Regularisation Methods"*
14.00 - 14.30	**Volker Michel** (Siegen, Germany): *"Seismic Tomography - Mathematical Modelling, Methods and Challenges"*
14.30 - 15.00	**Mourad Sini** (Linz, Austria): *"On the Accuracy of the Sampling Methods for Reconstructing Obstacles"*
15.00 - 15.30	**Break**
15.30 - 16.30	**Session 8, Chair: Sergei V. Pereverzyev (Linz, Austria)**
15.30 - 15.50	**Uno Hämarik** (Tartu, Estonia): *"Minimum Strategy for Parameter Choice in Regularization Methods"*
15.50 - 16.10	**Radu I. Boţ** (Chemnitz, Germany): *"Minimality via Convex Subdifferential Calculus: the Role of the Regularity Conditions"*
16.10 - 16.30	**Markus Grasmair** (Innsbruck, Austria): *"Restricted Isometry Property and Range Condition for Sparse Regularisation"*
16.30 - 16.50	**Break**

16.50 - 17.35	**Session 9, Chair: Antonio Leitão (Florianopolis, Brazil)**
16.50 - 17.05	**Marcus Meyer** (Chemnitz, Germany): *"Parameter Identification Problems for Elastic Large Deformations"*
17.05 - 17.20	**Thomas Bonesky** (Bremen, Germany): *"Regularization with Sparsity Constraints and Morozov's Discrepancy Principle"*
17.20 - 17.35	**Jens Geissler** (Chemnitz, Germany): *"Convergence Rates for Tikhonov Regularization with General Similarity Functional"*
17.35 - 17.45	**Closing and Conclusions**
18.00	**Conference Dinner in Restaurant 'Keplers'** Altenberger Straße 69 4040 Linz

Increasing Stability in Inverse Problems

Victor Isakov

Abstract: We discuss improving stability of the continuation for the Helmholtz type equations when frequency is increasing. We recall classical results of Fritz John showing that this phenomenon can happen only under some conditions and we give sufficient conditions in form of better stability estimates. Also we demonstrate increasing stability of the recovery of potential in the Schroedinger equation from the Dirichlet-to Neumann map. The proof use energy (in particular Carleman estimates), the Fourier analysis, complex geometrical optics, and properties of some fundamental solutions of general partial differential operators with constant coefficients.

Variational Models with Non-Gaussian Noise

Martin Burger

Abstract: In this talk we shall discuss some theoretical and computational issues for inverse problems with Non-Gaussian noise models such as Poisson, Laplace, or Multiplicative noise. Such models appear in several modern applications such as nanoscopy, PET, SPECT, and SAR.
Based on Bayesian MAP estimation we derive appropriate variational models, which we analyze with respect to convergence and convergence rates, highlighting the role of duality. Moreover, we discuss the construction of Bregman iterations and inverse scale space methods, which can be obtained in a primal as well as a dual version. Finally we comment on the construction of appropriate numerical schemes for such problems.
This talk is based on joint work with Martin Benning, Christoph Brune, Alex Sawatzky, Marzena Franek, Jahn Müller.

Some 'Probe-Methods' for the Reconstruction of Inverse Inclusion Problems

William Rundell

Abstract: We look at some computational schemes based on probing the interior region in order to determine an inclusion. This could be identified either by a fixed boundary condition or a different conductivity or internal source from the background medium. Unlike many similar schemes, we make the assumption that the provided data is not over-sampled but is in fact the minimum required for uniqueness of the inverse problem.

The Mathematical Model for the Diffusion Process and Related Inverse Problems

Jin Cheng

Abstract: In this talk, we will present some results about the diffusion process, especially for the abnormal diffusion. Some related applications are mentioned. It is shown that the mathematical results are useful for constructing the stable numerical algorithms and give more information for the engineers.

A Carleman Estimate and the Balancing Principle in the Quasi-Reversibility Method for Solving the Cauchy Problem for Laplace Equation

Sergei V. Pereverzyev

Abstract: The quasi-reversibility method to solve the Cauchy problem for the Laplace's equation in a bounded domain is considered. With the help of the Carleman estimations technique improved error and stability bounds in a subdomain are obtained. This paves a way for the use of the balancing principle for a posteriori choice of the regularization parameter in the quasi-reversibility method. As an adaptive regularization parameter choice strategy, the balancing principle does not require a priori knowledge of either the solution smoothness or a constant appearing in the stability bound estimation. Nevertheless, this principle allows an a posteriori parameter choice that up to controllable constant achieves the best accuracy guaranteed by the Carleman estimate.

Joint research with Hui Cao (Johann Radon Institute for Computational and Applied Mathematics) and Michael V Klibanov (Department of Mathematics and Statistics, University of North Carolina at Charlotte).

On some Minimization-Based Heuristic Parameter Choice in Inverse Problems

Peter Mathé

Abstract: The choice of the regularization parameter in inverse problems is most important, and it is the key to successfully solving such problems. Several parameter choice principles can be described as minimizers of certain functionals, among them the (modified) L-curve, the Hanke–Raus type, but also the discrepancy principles. In addition, the above mentioned rules use the discrepancy (together with other information) to judge the quality of a given regularization parameter. We provide a calculus for the analysis of such minimization based approaches, proving that these can enjoy optimality properties in many cases. The analysis is complemented with a discussion on how to actually carry out the minimization. We propose model functions and highlight that these yield efficient minimization procedures.

This study is joint work with Shuai Lu, RICAM, Linz: *Heuristic parameter selection based on functional minimization: optimality and model function approach*, 2009.

Inverse Problems for Diffusion Equations of Fractional Orders

Masahiro Yamamoto

Abstract: Recently diffusion equations of fractional orders are used as models for diffusion of underground contaminants. We discuss an inverse source problem and related inverse problems and show some results on the uniqueness.

Tikhonov Regularization with Sparsity Constraints - Regularization Properties, Convergence Rates and Optimization

Ronny Ramlau

Abstract: In this talk we consider the computation of a solution of an (possible nonlinear) operator equation $A(x) = y$ from noisy measurements y^δ. If A is an ill - posed operator, regularization methods have to be used in order to compute stable approximations to the solution. By choosing suitable bases or frames for the representation of functions belonging to $D(A)$ and $R(A)$, the operator equation can be considered between sequence spaces. We are in particular interested in the computation of sparse solutions, i.e., solutions that have only few nonzero entries with respect to the chosen basis or frame. To this end, we use Tikhonov regularization with a sparsity constraint (usually a weighted ℓ_p-norm with $0 < p < 2$).
In our talk we will present strategies for the computation of minimizers of the Tikhonov functionals under consideration. Additionally, we investigate a priori and a posteriori parameter choice rules $\alpha = \alpha(\delta)$ that ensure convergence (with respect to the underlying sparsity measure) of the minimizers x_α^δ of the Tikhonov functionals to a solution $x^\dagger$ of the equation. Assuming that the solution fulfills a source condition, we will also present convergence rates. Finally, we present numerical examples and applications that confirm our analytical results.

Iterative Regularization in Banach Spaces

Torsten Hein

Abstract: We consider the linear ill-posed operator equation

$$A\,x = y \qquad x \in X,\ y \in Y,$$

where $A : X \longrightarrow Y$ denotes a linear bounded operator mapping between the Banach spaces X and Y. For simplicity we assume A to be injective and there exists $x^\dagger \in X$ with $A\,x^\dagger = y$. For $\delta > 0$ and given noisy data $y^\delta \in Y$ with knowing bound $\|y^\delta - y\| \le \delta$ for the noise level we deal with iterative regularization approach

$$\begin{aligned} x_0^\delta &:= x_0 \in X,\ x_0^* := J_p(x_0^\delta) \\ x_{n+1}^* &:= x_n^* - \mu_n A^\star J_p(A\,x_n^\delta - y^\delta) - \beta_n x_n^\star \\ x_{n+1}^\delta &:= J_p^{-1}(x_{n+1}^*) \end{aligned}$$

together with a suitable (a posteriori) criterion for stopping the iteration. Here $p \in (1,\infty)$, $p^{-1} + q^{-1} = 1$ and J_p denotes the duality mapping from the space X (respectively Y) into its dual space X^* (respectively Y^*) with gauge function $t \mapsto t^{p-1}$.
In particular we discuss the following:

- We present two types of stopping criterions which terminate the algorithm after a finite number of iterations for all $\delta > 0$. The choice of the parameters μ_n and β_n depends on the underlying stopping criterion.
- We show $x_n^0 = x_n \to x^\dagger$ as $n \to \infty$.
- For one of the stopping criterion we derive an a priori convergence rates result based on the knowledge of an approximate source condition.
- We give some generalization of this method to nonlinear problems.

Finally we present a short numerical example.

An Extended State Approach to Online Parameter Identification in a Class of Infinite Dimensional Dynamical Systems

Philipp Kügler

Abstract: Online parameter identification for dynamical systems is the task of estimating model parameters simultaneously to the time evolution of the physical state and plays a crucial role in adaptive control. In this talk we present an extended state approach to online parameter identification in a class of time-dependent possibly nonlinear partial differential equations that works without evolutionary operator equations but still has the potential to allow for partial state observations.

Inverse Problems with Non-Convex Separable Constraints

Kristian Bredies

Abstract: A widely-used approach to solve ill-posed inverse problems is the minimization of appropriate Tikhonov functionals which usually consist of a discrepancy functional and a regularization term. It is often argued that the choice of the regularization term and the underlying space is important since they reflect model assumptions on the true solution. One famous example is the use of the sequence space $\ell 1$ and its norm in order to implement solutions which are sparse in a certain sense. For this regularization, many properties could be established, ranging from convergence rates to the realization of efficient minimization algorithms. A different view on sparsity is to consider the non-convex $\ell 0$ penalty which is counting the number of non-zero coefficients. In finite dimensions, it has been shown that in many interesting situations, $\ell 0$ and $\ell 1$ yield identical minimizers of the Tikhonov functional. This can, however, not be transferred to the infinite-dimensional case since, in general, $\ell 0$ does not lead to a well-posed minimization problem. Nevertheless, it is possible to obtain well-posedness by considering suitable approximating non-convex penalization functions.
In this talk, we introduce and study a framework for inverse problems with such non-convex constraints. On the one hand, the general theory is discussed, including well-posedness, regularization properties and convergence rates. As it turns out, convergence of order $\mathcal{O}(\delta)$ can often be established for the $\ell 1$-norm, in some cases even for the ℓ^p-quasi-norms when p is less than 1. Moreover, we present a simple and effective algorithm for the numerical minimization of the associated non-smooth and non-convex Tikhonov functionals. It is of generalized gradient projection type and implements the successive application of expansive, discontinuous thresholding functions. The convergence properties of this algorithm are discussed and some numerical examples are presented.
Joint work with Dirk Lorenz.

Multidimensional Inverse Heat Conduction Calculations

Hans-Jürgen Reinhardt

Abstract: Inverse Heat Conduction Problems (IHCPs) have been extensively studied over the last 50 years. They have numerous applications in many branches of science and technology. The problem consists in determining the temperature and heat flux at inaccessible parts of the boundary of a 2- or 3-dimensional body from corresponding data – called 'Cauchy data' – from accessible parts of the boundary. It is well-known that IHCPs are ill-posed which means that small perturbations in the data may cause large errors in the solution.

In this contribution we first present the problem and the associated adjoint problem in a variational formulation. The adjoint problem and the corresponding adjoint operator are extremely important for the iterative regularization of IHCPs by, e.g., the Conjugate Gradient Method.

We show examples of calculations for 2-dimensional IHCPs where the direct problems are solved with the Finite Element package DEAL.

Joint work with Jörg Frohne (University of Siegen).

Conditional Stability Estimates and Regularization with Applications to Cauchy Problems for the Helmholtz Equation

Ulrich Tautenhahn

Abstract: In this talk we consider the problem of reconstructing solutions $x^\dagger$ of ill-posed problems $Ax = y$ where A is a linear (not necessarily bounded) operator between Hilbert spaces X and Y with non-closed range $\mathcal{R}(A)$. We assume that instead of exact data $y \in \mathcal{R}(A)$ some noisy data $y^\delta \in Y$ with $\|y - y^\delta\| \leq \delta$ are given and that $x^\dagger$ possesses a certain solution smoothness which we describe by $x^\dagger \in M$ with some general source set $M \subset X$.

We discuss the following questions:

(i) Which best possible accuracy can be obtained for identifying $x^\dagger$ from noisy data $y^\delta \in Y$ under the assumptions $\|y - y^\delta\| \leq \delta$ and $x^\dagger \in M$?

(ii) How to regularize such that the best possible accuracy can be guaranteed?

We discuss cross-connections between conditional stability estimates and best possible error bounds for identifying $x^\dagger$ from noisy data under the two assumptions $\|y - y^\delta\| \leq \delta$ and $x^\dagger \in M$. We apply our results to Cauchy problems for the Helmholtz equation arising in optoelectronics, and in particular in laser beam models. We show that, depending on different smoothness situations, the best possible accuracy may be of Hölder type, of logarithmic type or of some other type. In addition, we study regularization methods that provide the best possible accuracy. In case of appropriate *a posteriori* parameter choice, the best possible order of accuracy can be obtained without using any smoothness information for $x^\dagger$.

This talk is a joint work with Teresa Regińska, Institute of Mathematics, Polish Academy of Sciences, Warsaw, Poland.

On Multiple Level Set Type Regularization Methods

Antonio Leitão

Abstract: We analyze regularization methods for solving inverse problems where the sought surface in 2D or 3D is piecewise constant with several possible level values. These levels may further be potentially unknown. Multiple level set functions are used to represent the solution when there are more than two such levels.

Starting from a multiple level-set approach, a Tikhonov functional based on TV-$H1$ penalization is proposed. We define generalized minimizers for this Tikhonov functional and establish an existence result. Moreover, we prove convergence and stability results for the proposed Tikhonov method.

A rather efficient multiple level-set method is developed from the first order optimality conditions for the Tikhonov functional. Several inverse potential problems in two and three space variables are solved numerically, demonstrating the method's capabilities for both known and unknown level values.

A Bayesian Model for Root Computation

Hanna K. Pikkarainen

Abstract: We study the root computation problem where a univariate polynomial over the real or the complex numbers is given approximately. We present a Bayesian method for the computation of the posterior probabilities of different multiplicity patterns. The method is based on interpreting the root computation problem as an inverse problem which is then treated in the Bayesian framework. The performance of the method is illustrated by numerical examples.

New Uniqueness Results for Parabolic Integro-Differential Identification Problems

Jaan Janno

Abstract: We present uniqueness results for two identification problems for parabolic integro-differential equations (PIDE) containing convolutions over time with kernels h. These results essentially use extremum principles.
The first problem is to determine an unknown x-dependent (i.e. space-dependent) factor of a source term by means of final over-determination of the solution of PIDE. We prove a positivity principle for PIDE with scalar kernels and use this principle to establish the uniqueness for the inverse problem. As a corollary, the uniqueness for inverse problems to determine unknown x-dependent coefficients of PIDE follows, too.
Secondly, we consider an inverse transmission problem for PIDE in an open domain Ω where the subdomains of continuity are Ω_1 and $\Omega_2 = \Omega \setminus \overline{\Omega}_1$ such that $\partial\Omega_2 = \partial\Omega_1 \cup \partial\Omega$ and $\mathrm{dist}\{\partial\Omega_1, \partial\Omega\} > 0$. The restrictions of h in the subdomains Ω_1 and Ω_2 are assumed to be scalar and denoted by h_1 and h_2, respectively. The inverse problem consists in determination of h_1 from measured flux in $\partial\Omega$ over the time. This problem is severely ill-posed, because it reduces to an equation with an infinitely smoothing operator. In case the problem is formulated in the infinite time interval $(0, \infty)$, we apply the Laplace transform and use the extremum principle and Giraud theorem for the resulting elliptic problem to prove the uniqueness for the inverse problem. The uniqueness in the finite time interval $(0, T)$ is an open problem in the moment of submission of this abstract, as it requires an extremum principle for PIDE with non-scalar h.

Data Assimilation in Numerical Weather Prediction: 4D-Var and Links to Other Regularisation Methods

Melina Freitag

Abstract: In this talk we will give an introduction to data assimilation techniques as they are used in modern numerical weather prediction. It is well-known that data assimilation using 4D-Var (4D Variation) can be interpreted as some form of Tikhonov regularisation, a very familiar method for solving ill-posed inverse problems. Such problems appear in a wide range of applications such as geosciences and image restoration, the process of estimating an original image from a given blurred image.
From the latter work it is known that by replacing the L_2-norm penalty function by an L_1-norm penalty function the image restoration problems become edge-preserving as they do not penalise the edges of the image. The L_1-norm penalty regularisation then recovers sharp edges in the image better than the L_2-norm penalty regularisation. We apply this idea to 4D-Var for problems where shocks are present and give some examples where the L_1 norm penalty approach performs much better than the standard L_2 norm regularisation in 4D-Var.
This work is supported by GWR (Great Western Research) and the UK MetOffice and joint with N. Nichols (Reading) and C. Budd (Bath).

Seismic Tomography - Mathematical Modelling, Methods and Challenges

Volker Michel

Abstract: Earthquake data reveal a lot of information about the Earth's interior. Typical data are the travel-times of seismic waves and the frequency anomalies of free oscillations after major earthquakes. In the lecture, a brief insight into the mathematical modelling of the associated inverse problems is given, properties of the problems (such as the aspects of ill-posedness) are discussed, and new mathematical methods using localised basis functions are demonstrated.

References:

1. A. Amirbekyan, V. Michel: Splines on the three-dimensional ball and their application to seismic body wave tomography, Inverse Problems, 24 (2008), http://stacks.iop.org/0266-5611/24/015022, doi:10.1088/0266-5611/24/1/015022.

2. A. Amirbekyan, V. Michel, F.J. Simons: Parameterizing surface-wave tomographic models with harmonic spherical splines, Geophysical Journal International, 174 (2008), 617-628, http://www3.interscience.wiley.com/journal/120735399/abstract.

3. P. Berkel, V.Michel: On Mathematical Aspects of a Combined Inversion of Gravity and Normal Mode Variations by a Spline Method, Schriften zur Funktionalanalysis und Geomathematik, 41 (2008), Preprint, Fachbereich Mathematik, TU Kaiserslautern.

On the Accuracy of the Sampling Methods for Reconstructing Obstacles

Mourad Sini

Abstract: In this talk, we deal with the acoustic inverse scattering problem for detecting obstacles from exterior measurements.
In the last years, several non-iterative methods have been proposed to reconstruct the surface of the obstacles using exterior measurements. Among others, we can cite the linear sampling method, the factorization method, the music algorithms and the probe methods. These methods are based on building up indicator functions which behaviors change drastically near the interface of the obstacle. This change in the behavior is used to set up numerical algorithms to detect the obstacles.
In this talk, we give some details on the behavior of those indicator functions and show how the geometry of the obstacle has influence on the accuracy of the reconstruction. We also provide some numerical examples showing this dependency. In addition, we will discuss some ways of increasing the accuracy by 1.) using high frequencies of incidence and 2.) using surface coatings to decrease the effect of the geometry.

Minimum Strategy for Parameter Choice in Regularization Methods

Uno Hämarik

Abstract: We consider ill-posed problem $Au = f$, where A is a linear continuous operator acting between Hilbert spaces, and instead of f, noisy data f_δ with $\|f_\delta - f\| \le \delta$ are given. We study choice of the regularization parameter α in Tikhonov method $u_\alpha = (\alpha I + A^*A)^{-1}A^*f_\delta$ from α-sequence $\alpha_i = q^i$, $i = 0, 1, \ldots$ $(q < 1)$. We use also extrapolated approximations $v_{2,\alpha_i} = (1-q)^{-1}(u_{\alpha_i} - qu_{\alpha_{i-1}})$, $v_{3,\alpha_i} = (1-q)^{-2}[(1+q)^{-1}u_{\alpha_{i-1}} - qu_{\alpha_i} + q3(1+q)^{-1}u_{\alpha_{i+1}}]$, and $v_{4,\alpha_i} = (1-q)^{-3}(1+q)^{-1}((1+q+q2)^{-1}u_{\alpha_{i-2}} - qu_{\alpha_{i-1}} + q3u_{\alpha_i} - q6(1+q+q2)^{-1}u_{\alpha_{i+1}})$, see [1]. Denote $r_{1,i} := Au_{\alpha_i} - f_\delta$, $r_{k,i} := Av_{k,\alpha_i} - f_\delta$. Let $m = 1$ for approximation u_α, and $m = 2$ for approximation $v_{2,\alpha}$. In discrepancy principle, in monotone error rule and in the rule R2 (see [3]) parameters α_{D}, α_{ME} and α_{R2} in these approximations are chosen as the largest α_i for which $\|r_{m,i}\| \le \delta$, $(r_{m,i}, r_{m+1,i})/\|r_{m+1,i}\| \le \delta$ or $(\alpha_i^{-1} + \|A\|^{-2})^{1/2}\|A^*r_{m+1,i}\|^2(A^*r_{m+1,i}, A^*r_{m+2,i})^{-1/2} \le b_m\delta$, respectively, with $b_1 = 0.3$, $b_2 = 0.2$. We propose to choose for the regularization parameter the minimum of some expressions of arguments α_{ME} and α_{R2}. Examples: parameters $\alpha_{\mathrm{M1}} = \min(0.5\alpha_{\mathrm{ME}}, 0.6\alpha_{\mathrm{ME}}^{1.06})$, $\alpha_{\mathrm{M2}} = \min(\alpha_{\mathrm{M1}}, \alpha_{\mathrm{R2e}})$ with $\alpha_{\mathrm{R2e}} = 0.5\alpha_{\mathrm{R2}}$ are good in cases $\|f - f_\delta\| = \delta$, $\|f - f_\delta\| \le \delta$, respectively. We solved test problems from [2] (10 problems), using besides solution u_* also smoother solution $u_{*,p} = (A^*A)^{p/2}u_*$ with $f = Au_{*,p}$. We used $p = 2$ and $q = 0.9$. The problems were normalized, so that norms of the operator and the right hand side were 1. For perturbed data we took $f_\delta = f + \Delta$, $\|\Delta\| = 0.5$, 10^{-1}, $\ldots$, 10^{-6} with 10 different perturbations Δ generated by computer.
For used rules we present averages and maximums of error ratios $\|u_\alpha - u_*\|/e_{\mathrm{opt}}$ in case $\delta = d\|f_\delta - f\|$, where $e_{\mathrm{opt}} = \min\{\|u_\alpha - u_*\| : \alpha \ge 0\}$ and $d = 1, 1.3, 2$. In case $d = 0.7$ other rules except R2 and R2e fail but the rule R2e gave averages 1.45 and 1.17 for cases $p = 0$ and $p = 2$, respectively. Columns vR2e, vM1, vM2 show averages and maximums of error ratios $\|v_{2,\alpha} - u_*\|/e_{\mathrm{opt}}$ for rules R2e, M1, M2.

		Averages							Maximums						
d	p	D	R2e	M1	M2	vR2e	vM1	vM2	D	R2e	M1	M2	vR2e	vM1	vM2
1	0	1.20	1.38	1.16	1.16	1.42	1.15	1.16	4.6	15	3.7	3.3	18	4.8	4.8
1	2	2.70	1.11	1.10	1.11	0.67	0.63	0.65	25	3.8	2.8	2.1	5.9	3.5	4.1
1.3	0	1.86	1.46	1.72	1.44	1.46	1.67	1.44	19	16	19	16	19	20	19
1.3	2	2.40	1.11	2.14	1.11	0.66	1.17	0.66	26	5.0	19	5.0	6.3	12	6.3
2	0	2.43	1.59	2.23	1.58	1.58	2.19	1.57	34	17	31	17	19	30	19
2	2	3.37	1.20	3.62	1.20	0.70	2.46	0.70	61	7.9	51	7.9	6.8	49	6.8

Joint work with R. Palm and T. Raus (University of Tartu, Estonia).

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Minimality via Convex Subdifferential Calculus: the Role of the Regularity Conditions

Radu I. Boţ

Abstract: In this talk we outline the role of the regularity conditions when characterizing minimality by means of convex subdifferential calculus. The focus is posed on the generalized interior-point conditions, but the so-called closedness-type ones are mentioned, too. On the one hand, some related results given in the literature in the context of convex regularization approaches are discussed. On the other hand, we offer a glimpse into the importance of the regularity conditions when dealing with maximal monotone operators.

Restricted Isometry Property and Range Condition for Sparse Regularisation

Markus Grasmair

Abstract: Sparse regularisation, that is, regularisation of an operator equation $Fx = y$ with an $\ell 1$ penalty term, has received much attention during the last years. From a theoretical point of view, one of the most interesting results is that sparse regularisation allows convergence rates of order $O(\delta)$. Such rates have been derived using two different types of conditions: first, a range condition together with the finite basis injectivity property, which postulates injectivity of F on certain finite dimensional subspaces, and second, the restricted isometry property, which requires the restrictions of F to low dimensional subspaces to be almost isometric. In this talk we discuss the relations between these apparently different conditions.

Parameter Identification Problems for Elastic Large Deformations

Marcus Meyer

Abstract: In the field of mechanical engineering the simulation of the deformation of bodies induced by loads (e.g. forces and moments) plays an essential role. From the inverse point of view, the simulation denotes a forward problem, where the material behavior is described by material laws and the displacement of the body has to be found. In general, material models include several model parameters representing some material properties. For the application of simulation tools these material parameters have to be known, which leads to inverse parameter identification problems. Due to the fact, that the model parameters are not a priori known, they have to be estimated by indirect measurements.
In this presentation we consider elastic deformation problems with large deformations. In contradiction to the well known linear theory for small deformations, we here have to deal with problems, where the displacement is found as the solution of a nonlinear PDE. For a repertory of material laws, we give a survey on the implementation of direct and inverse problems. Solution approaches for the parameter identification problems are presented, using multi-parameter regularization, Lagrange- and SQP-methods. The identification of material parameters with experimental data is discussed.

Regularization with Sparsity Constraints and Morozov's Discrepancy Principle

Thomas Bonesky

Abstract: Regularization with sparsity constraints is an important field of research in inverse problems. However, regularization properties and convergence rates for sparsity reconstruction are mainly based on a priori parameter choice strategies. The investigation of an a posteriori parameter choice rule was the motivation for the presented work. The talk deals with the combination of Morozov's discrepancy principle and Tikhonov regularization considering functionals of the form

$$\Gamma_{\alpha,\delta}(x) = \|Ax - y^{\delta}\|^2 + \alpha\Omega(x),$$

where the classical L_2-penalty term is substituted by a more general functional Ω. This functional may especially be a sparsity enforcing penalty term. We show regularizing properties for the combined scheme as well as convergence rates. Supported by Deutsche Forschungsgemeinschaft, grant number MA 1657/15-1.

Convergence Rates for Tikhonov Regularization with General Similarity Functional

Jens Geissler

Abstract: Regularization of nonlinear inverse and ill-posed problems by minimizing a so-called Tikhonov functional, which is the weighted sum of a similarity functional for approximating given data and a regularization functional for stabilizing the problem, recently has been extended in two directions: On the one hand, instead of using the norm of a Banach space, general similarity functionals were introduced. On the other hand convergence rates results were extended to so-called approximate source conditions. This talk combines these two aspects by formulating approximate source conditions for Tikhonov regularization with general similarity functional.

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