TECHNISCHE UNIVERSITÄT CHEMNITZ

Department of Mathematics
Analysis – Inverse Problems

Chemnitz Symposium on Inverse Problems 2014

Conference Guide

September 18 – 19, 2014

Chemnitz, Germany

General information
Timetable
Abstracts
List of participants

Imprint

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General information

Goal

Our symposium will bring together experts from the German and international ‘Inverse Problems Community’ and young scientists. The focus will be on ill-posedness phenomena, regularization theory and practice, and on the analytical, numerical, and stochastic treatment of applied inverse problems in natural sciences, engineering, and finance.

Location

Technische Universität Chemnitz
Straße der Nationen 62 (Böttcher-Bau)
Conference hall ‘Altes Heizhaus’
09111 Chemnitz, Germany

Selection of invited speakers

Andrew M. Stuart (Warwick, Great Britain)
Elena Resmerita (Klagenfurt, Austria)
Bastian von Harrach (Stuttgart, Germany)
Markus Grasmair (Trondheim, Norway)

Scientific board

Bernd Hofmann (Chemnitz, Germany)
Peter Mathé (Berlin, Germany)
Sergei V. Pereverzyev (Linz, Austria)
Masahiro Yamamoto (Tokyo, Japan)

Organizing committee

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Timetable

Overview for Thursday, September 18

09.00–09.05	**Introduction**
09.05–10.40	**Session 1** A. Stuart, P. Mathé, E, Resmerita
10.40–11.00	**Coffee break**
11.00–12.15	**Session 2** B. Harrach, M. Grasmair, V. Michel
12.15–13.15	**Lunch break**
13.15–14.55	**Session 3** M. Yamamoto, C. Clason, F. Werner, R. Kowar
14.55–15.15	**Coffee break**
15.15–16.55	**Session 4** T. Regińska, M. Schlottbom, C. Gerhards, I. R. Bleyer
17.10–21.30	**Excursion**

Overview for Friday, September 19

09.00–09.10	**Laudation**
09.10–10.25	**Session 1** S. V. Pereverzyev, M. Hegland, S. W. Anzengruber
10.25–10.45	**Coffee break**
10.45–12.25	**Session 2** S. Hollborn, T. Helin, J.-F. Pietschmann, S. Pereverzyev Jr.
12.25–13.20	**Lunch break**
13.20–14.35	**Session 3** H. Kekkonen, S. S. Nanthakumar, F. Margotti, R. Winkler, S. Orzlowski
14.35–14.45	**Coffee break**
14.45–16.00	**Session 4** Z. Purisha, P. Tkachenko, D. Saxenhuber, D. Gerth, S. Bürger

Program for Thursday, September 18

09.00–09.05	**Introduction**
09.05–09.50	Opening lecture **Andrew Stuart** (Warwick, Great Britain) *Well-posed Bayesian geometric inverse problems*
09.50–10.15	**Peter Mathé** (Berlin, Germany) *Merging regularization theory into Bayesian inverse problems*
10.15–10.40	**Elena Resmerita** (Klagenfurt, Austria) *An entropic Landweber-type method for linear ill-posed problems*
10.40–11.00	**Coffee break**
11.00–11.25	**Bastian von Harrach** (Stuttgart, Germany) *Inverse coefficient problems and shape reconstruction*
11.25–11.50	**Markus Grasmair** (Trondheim, Norway) *Bregman distance, source conditions, and variational inequalities in Tikhonov regularisation*
11.50–12.15	**Volker Michel** (Siegen ,Germany) *The regularized (orthogonal) functional matching pursuit - a best basis algorithm for inverse problems in geomathematics and medical imaging*
12.15–13.15	**Lunch break**
13.15–13.40	**Masahiro Yamamoto** (Tokyo, Japan) *Inverse problems of moving sources in wave equation*
13.40–14.05	**Christian Clason** (Essen, Germany) *Stochastic inverse problems with impulsive noise*

14.05–14.30	**Frank Werner** (Göttingen, Germany) *Statistical inverse problems in fluorescence microscopy*
14.30–14.55	**Richard Kowar** (Innsbruck, Austria) *On time reversal in photoacoustic tomography for tissue similar to water*
14.55–15.15	**Coffee break**
15.15–15.40	**Teresa Reginska** (Warsaw, Poland) *Solution-functional and data-functional regularization strategies for determining the laser beam quality parameters*
15.40–16.05	**Matthias Schlottbom** (Münster, Germany) *Identification of nonlinear heat conduction laws in heat transfer problems*
16.05–16.30	**Christian Gerhards** (Siegen, Germany) *Combining downward continuation and local approximation on different spheres by optimized kernels*
16.30–16.55	**Ismael Rodrigo Bleyer** (Helsinki, Finland) *Digital speech: an application of the dbl-RTLS method for solving GIF problem*
17.10–21.30	Excursion to Wasserschloss Klaffenbach and conference dinner

Time periods include 5 minutes for discussion.

Program for Friday, September 19

The first morning session is dedicated to the 60th anniversary of PD Dr. Peter Mathé.

09.00–09.10	**Bernd Hofmann** (Chemnitz, Germany) *Laudation*
09.10–09.35	**Sergei V. Pereverzyev** (Linz, Austria) *Aggregation of regularized approximations*
09.35–10.00	**Markus Hegland** (Canberra, Australia) *Weighted function spaces for highdimensional approximation and multiparameter regularisation*
10.00–10.25	**Stephan W. Anzengruber** (Chemnitz, Germany) *A NURBS-based gradient method for sparse angle tomography*
10.25–10.45	**Coffee break**
10.45–11.10	**Stefanie Hollborn** (Mainz, Germany) *Backscatter data in electric impedance tomography*
11.10–11.35	**Tapio Helin** (Helsinki, Finland) *Inverse scattering in half-space with random boundary conditions*
11.35–12.00	**Jan-Frederik Pietschmann** (Darmstadt, Germany) *Identification of chemotaxis models with volume filling*
12.00–12.25	**Sergiy Pereverzyev Jr.** (Innsbruck, Austria) *Multi-penalty regularization for detecting relevant variables*
12.25–13.20	**Lunch break**
13.20–13.35	**Hanne Kekkonen** (Helsinki, Finland) *White noise paradox in Bayesian inverse problems*

13.35–13.50	**Srivilliputtur Subbiah Nanthakumar** (Weimar, Germany) *Inverse problem of multiple inclusions detection in piezoelectric structures using XFEM and Level sets*
13.50–14.05	**Fabio Margotti** (Karlsruhe, Germany) *Inexact newton regularization methods in Banach spaces*
14.05–14.20	**Robert Winkler** (Karlsruhe, Germany) *Adaptive sensitivity-based regularization for Newton-type inversion in electrical impedance tomography*
14.20–14.35	**Sarah Orzlowski** (Siegen, Germany) *Regularized joint inversion of EEG and MEG data by a best basis algorithm*
14.35–14.45	**Coffee break**
14.45–15.00	**Zenith Purisha** (Helsinki, Finland) *Recovering shape of 2D pipe with corrosion and attenuation coefficient with limited data*
15.00–15.15	**Pavlo Tkachenko** (Linz, Austria) *Multi-parameter regularization of ill-posed spherical pseudo-differential equations in C-space*
15.15–15.30	**Daniela Saxenhuber** (Linz, Austria) *Atmospheric tomography for ELT adaptive optics*
15.30–15.45	**Daniel Gerth** (Linz, Austria) *The method of the approximate inverse for atmospheric tomography*
15.45–16.00	**Steven Bürger** (Chemnitz, Germany) *An autoconvolution problem connected with SD-SPIDER*

Time periods include 5 minutes for discussion.

Abstracts

A NURBS-based gradient method for sparse angle tomography

Stephan W. Anzengruber

In sparse angle tomography one attempts to reconstruct an image from highly undersampled Radon data. The resulting severely ill-posed problem becomes more stable, if the solution is known to be piecewise constant. This has been used in Mumford-Shah-type regularization methods [2, 3], for example, but recently also in [1] where the interfaces in the image are parametrized by NURBS (Non-Uniform Rational B-splines). The latter approach even yields reconstructions in vector-graphics format, but the dependence of the image on the design parameters of the interface curves is non-linear and not differentiable.

In this talk we will see that Radon data, however, remain differentiable almost everywhere and then use gradient-type methods to find NURBS design parameters for the interface curves from sparse angle tomographic data.

References

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Digital speech: an application of the dbl-RTLS method for solving GIF problem

Ismael Rodrigo Bleyer

"Digital Speech Processing" refers to the study of a speech signal. Namely, these signals are processed in a digital representation, as for example, synthesis, analysis, enhancement, compression and recognition may refers to this process.

In this talk we are interested on solving the core problem known as "Glottal Inverse Filtering" (GIF). Commonly this problem can be modelled by convolving a pressure function (input signal) with an impulse response function (filter). Our approach is done in a deterministic setup based on the dbl-RTLS (double regularised total least squares) approach introduced recently. Additionally we review briefly the methodology and we give the first numerical realisations, based on an alternating minimisation procedure.

Keywords

ill-posed problems, noisy operator, noisy right hand side, regularised total least squares, double regularisation, alternating minimisation, glottal inverse filtering, digital speech

An autoconvolution problem connected with SD-SPIDER

Steven Bürger

In non-linear optics SD-SPIDER is a proposed method to measure ultra-short laser pulses. For the reconstruction of the electric field of the pulses from the measured data one needs to solve an autoconvolution problem. For this problem we present theoretical results and numerical examples with artificial data.

Stochastic inverse problems with impulsive noise

Christian Clason

Impulsive noise models such as salt & pepper noise are frequently used in mathematical image restoration, but are usually restricted to the discrete setting. We introduce an infinite-dimensional stochastic impulsive noise model based on marked Poisson point processes, discuss its properties and show regularization properties of inverse problems subject to such noise. We also treat the conforming discretization of such noise models and compare this to the classical discrete impulsive noise. This is joint work with Laurent Demaret (Helmholtz-Zentrum München).

Combining Downward Continuation and Local Approximation on Different Spheres by Optimized Kernels

Christian Gerhards

Various problems in gravitation and geomagnetism (that are based on harmonic potentials) require the combination of satellite data and ground data at (or near) the Earth's surface in order to obtain high resolution models. While satellite data is available globally and requires ill-posed downward continuation to the Earth's surface, ground data does not suffer from this ill-posedness but it is only available locally/regionally. Therefore, satellite data is suitable for the reconstruction of large-scale features, whereas local/regional ground data can be used for the detection of spatially finer structures.

In this talk, we present a multiscale-based method that pays tribute to the different properties of the two types of data. More precisely, the approximation is based on a two-step algorithm: First, a convolution with a scaling kernel Φ_N deals with the downward continuation from the satellite orbit Ω_R to the spherical Earth's surface Ω_r, $r < R$, while in a second step, the result is locally refined by a convolution in a subregion $\Gamma_r \subset \Omega_r$ with a wavelet kernel $\tilde{\Psi}_N$. The kernels Φ_N and $\tilde{\Psi}_N$ are simultaneously optimized in such a way that the former behaves well for the downward continuation while the latter shows a good localization in Γ_r. The approach is indicated for a scalar and vectorial setting. Furthermore, numerical tests and comparisons to existing methods are presented.

The method of the approximate inverse for atmospheric tomography

Daniel Gerth, Bernadette Hahn, Ronny Ramlau

The method of the approximate inverse is a regularization scheme to determine a stable solution of an ill-posed operator equation. Instead of the original one, an auxiliary problem has to be solved to compute the so called reconstruction kernels, which are then used to reconstruct a mollified version of the desired function at certain grid points with the help of the measured data. Using invariances, all individual reconstruction kernels can easily be calculated from the solution of a single auxiliary problem that may be precomputed, hence saving computational effort. We apply this idea to the problem of atmospheric tomography, used for large ground-based telescopes which rely on adaptive optics systems to remove the effects of atmospheric distortions in order to achieve a good image quality. Adaptive optics systems physically correct atmospheric turbulences via deformable mirrors in real-time. The optimal shape of the deformable mirrors is determined from wavefront measurements of natural guide stars as well as laser guide stars. Numerical results will be demonstrated for the European Extremely Large Telescope (E-ELT).

Bregman distance, source conditions, and variational inequalities in Tikhonov regularisation

Markus Grasmair

In classical Tikhonov regularisation for the stable solution of a linear operator equation $Fu = v^\delta$ on a Hilbert space, it is possible to derive a range of convergence rates if the true solution $u^\dagger$ satisfies a source condition of the form $u^\dagger = (F^*F)^\nu\omega$ with $0 < \nu \leq 1$. The main step in the derivation of these rates is the formulation of an interpolation inequality involving the similarity term and the regularisation term; for the higher order rates obtained with $1/2 < \nu \leq 1$, this may be done using a dual formulation of the Tikhonov functional. In the more general setting of regularisation on Hilbert spaces with a non-quadratic regularisation term, this approach can be used for obtaining convergence rates with respect to the Bregman distance, which measures the deviance of the regularisation term from an affine approximation. The rates obtained in that manner, however, are for $\nu < 1/2$ typically of lower order than the rates with comparable conditions for quadratic regularisation. Higher order rates can be obtained though, if the norm can be estimated from above by the Bregman distance. Another possibility for deriving convergence rates is the usage of variational inequalities involving the similarity term and the Bregman distance. In the quadratic setting, these conditions almost provide a characterisation of the range of $(F^*F)^\nu$. In this talk we will discuss the differences and limitations of these two approaches. In particular, we will focus on the setting of total variation regularisation and provide some geometrical interpretation of the results in this situation.

Inverse coefficient problems and shape reconstruction

Bastian Harrach

Novel imaging methods commonly lead to the inverse problem of determining one or several coefficient function(s) in a partial differential equation from (partial) knowledge of its solutions. Arguably, the most famous such problem is the Calderón problem, where the diffusion coefficient in an elliptic PDE is to be reconstructed from the Neumann- and Dirichlet-boundary data of its solutions.

The mathematical challenges behind inverse coefficient problems reach from theoretical uniqueness questions to the construction of convergent numerical algorithms and stability issues. In this talk we will present recent work on these subjects that is based on combining monotony relations between the coefficients and the Neumann/Dirichlet-data with so-called localized potentials. We will show how to derive simple uniqueness results, avoid non-linearity effects in the context of shape-reconstruction, and deal with imprecisely known background media and measurement noise.

Weighted function spaces for highdimensional approximation and multiparameter regularisation

Markus Hegland

In a recent paper [J.Complexity,29(1),p.76-91,2013] with Greg Wasilkowski, spaces of multivariate functions were studied in which approximation does not suffer under the curse of dimensionality. Originally, these function spaces were used in data mining to show convergence of ANOVA decompositions. The arguments used relate to the concentration of measure.

It was recently suggested to the author that the these function spaces – which are reproducing kernel Hilbert spaces – may also be used for inverse problems and in particular provide a framework for multiparameter regularisation.

In this talk I will review the original results on approximation and then discuss opportunities for multivariate and high-dimensional regularisation.

Inverse scattering in half-space with random boundary conditions

Tapio Helin

An inverse acoustic scattering problem in half-space is studied. We assume an impedance boundary value condition, where the Robin coefficient is a Gaussian random function λ. The covariance operator of λ is assumed to be a classical pseudodifferential operator. Our data is the amplitude of the backscattered field averaged over the frequency band. We assume that such data is generated by a single realization of the random Robin coefficient. It is shown that in such a case the principal symbol of the covariance operator of λ is uniquely determined. We have an anisotropic stochastic model for the principal symbol, which leads to the analysis of a novel anisotropic spherical Radon transform and its invertibility. This is joint work with Matti Lassas and Lassi Päivärinta.

Backscatter Data in Electric Impedance Tomography

Stefanie Hollborn

This talk presents the concept of backscatter data in electric impedance tomography. These sparse data are the analogue to so-called backscattering data in inverse scattering. They arise in practice if the same single pair of electrodes is used to drive currents and measure voltage differences subsequently at various locations on the boundary of the object to be imaged. A single insulating cavity within an otherwise homogeneous object is uniquely determined by its backscatter data. However, this is in general not true for a perfectly conducting inclusion. The talk presents the uniqueness proof for an insulating cavity, and it outlines a reconstruction algorithm based thereon. The results presented here arise from joint research with Martin Hanke and Nuutti Hyvnen.

White noise paradox in Bayesian inverse problems

Hanne Kekkonen, Matti Lassas, Samuli Siltanen

Our aim is to provide new analytic insight to the relationship between the continuous and practical inversion models corrupted by white Gaussian noise.

Let us consider an indirect noisy measurement m of a physical quantity u

$$m(x) = Au(x) + \delta\varepsilon(x),$$

where $\delta > 0$ is the noise magnitude. If ε was an L^2-function, Tikhonov regularization gives an estimate

$$T_\alpha(m) = \operatorname*{argmin}_{u \in H^r} \left\{ \|Au - m\|_{L^2}^2 + \alpha \|u\|_{H_r}^2 \right\}$$

for u where $\alpha = \alpha(\delta)$ is the regularization parameter. Here penalization of the Sobolev norm $\|u\|_{H^r}$ covers the cases of standard Tikhonov regularization ($r = 0$) and first derivative penalty ($r = 1$).

Realizations of white Gaussian noise are almost never in L^2, but do belong to H^s with probability one if $s < 0$ is small enough. That is why we present a modification of Tikhonov regularization theory covering the case of white Gaussian measurement noise. We will also consider the question in which space does the estimate convergence to a correct solution when the noise variance goes to zero and what is the speed of the convergence.

The results are important for removing the apparent paradox arising from the use of Tikhonov regularization in discrete case and the infinite L^2-norm of the natural limit of white Gaussian noise in $\mathbb{R}^n$ as $n \to \infty$. Rigorous connection between discrete and infinite-dimensional limit model is often useful. Examples include error analysis for numerical inversion and computational speed-ups based on consistent switching between different discretizations related to multigrid methods.

On time reversal in photoacoustic tomography for tissue similar to water

Richard Kowar

This paper is concerned with time reversal in *photoacoustic tomography* (PAT) of dissipative media that are similar to water. Under an appropriate condition, it is shown that the time reversal method in [2, 1] based on the non-causal thermo-viscous wave equation can be used if the non-causal data is replaced by a *time shifted* set of causal data.

We investigate a similar imaging functional for time reversal and an operator equation with the time reversal image as right hand side. If required, an enhanced image can be obtained by solving this operator equation. Although time reversal (for noise-free data) does not lead to the exact initial pressure function, the theoretical and numerical results of this paper show that regularized time reversal in dissipative media similar to water is a valuable method. We note that the presented time reversal method can be considered as an alternative to the causal approach in [?] and a similar operator equation may hold for their approach.

Keywords

time reversal, photoacoustic tomography, inverse and ill-posed problems

AMS subject classifications

35R30, 35L05, 47A52, 45A05, 65J20

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Inexact Newton regularization methods in Banach spaces

Fábio Margotti, Andreas Rieder

Inexact Newton methods have proven to be a powerful class of iterative methods to solve nonlinear inverse and ill-posed problems in Hilbert spaces. To realize such a method one must linearize the equation around the current iteration and then apply a regularization technique to solve the resulting linear system. We propose here the adaptation of some classical regularization methods of gradient- and Tikhonov-type to solve the linearized system in the Banach spaces setting.

Merging regularization theory into Bayesian inverse problems

Peter Mathé

We discuss Bayesian linear inverse problems

$$y^\delta = Kx + \delta\eta$$

in Hilbert space with Gaussian noise η and the noise level δ. Under a Gaussian prior for the unknown solution x, say with mean m_0 and covariance C_α^δ, depending on the noise level δ, and an additional tuning parameter α, the posterior distribution, given the data y^δ will also be Gaussian. The focus is on a fast concentration of the posterior probability around the unknown true solution as expressed in the concept of posterior contraction rates (as $\delta \to 0$).

As previous analysis on posterior contraction rates revealed, this can be achieved by a proper choice of α, depending on the solution smoothness (a priori choice) or adaptive (oracle-type, a posteriori choice). We refer to B. T. Knapik, A. W. van der Vaart, and J. H. van Zanten, *Bayesian Inverse Problems with Gaussian Prior*, Ann. Statist. (39), 2011, Theorem 4.1, for details and a discussion. In particular these authors discuss the *saturation* of the contraction rate. Since then this phenomenon was also observed in other studies. Heuristically, this saturation can be explained by relating the Bayesian approach to Tikhonov regularization. The question is whether and how to overcome this resistance of the posterior contraction rates to higher smoothness.

It will be shown that this actually can be achieved by centering the prior distribution appropriately, i.e., with a choice for the prior mean $m_0 := m_0(\alpha, y^\delta)$. The idea and the analysis use results from regularization theory. This is joint work with S. Agapiou, University of Warwick.

The Regularized (Orthogonal) Functional Matching Pursuit — a Best Basis Algorithm for Inverse Problems in Geomathematics and Medical Imaging

Volker Michel

We present a novel algorithm, the RFMP, and its enhancement, the ROFMP, for the regularization of ill-posed inverse problems of the following kind: Given $y \in \mathbb{R}^l$ and linear and continuous functionals $\mathcal{F}^1, \ldots, \mathcal{F}^l : \mathcal{H}(D) \to \mathbb{R}$, find $f \in \mathcal{H}(D)$ such that $\mathcal{F}^j f = y_j$ for all $j = 1, \ldots, l$. Here, $\mathcal{H}(D)$ is a certain Hilbert space of functions on D, e.g. an L^2-space, a Sobolev space, or a reproducing kernel Hilbert space. Examples of such problems occur in geomathematics and medical imaging, e.g. for the evaluation of satellite data of the Earth, the exploration of the Earth's interior, the observation of climate-based water mass transports, and the localization of particular brain activities.

The ROFMP and the RFMP are improvements of methods by Mallat & Zhang, by Vincent & Bengio, and by Pati et al. which were developed for data interpolation without a regularization. The basic idea is as follows: We initially define a so-called dictionary $\mathcal{D} \subset \mathcal{H}(D)$ which consists of trial functions of different kinds which might be useful for the problem to be solved. Then, we iteratively construct a sequence of approximations in the sense that $F_{n+1} := F_n + \alpha_{n+1} d_{n+1}$, $F_0 := 0$, and $\alpha_{n+1} \in \mathbb{R}$ as well as $d_{n+1} \in \mathcal{D}$ are chosen such that the regularized data misfit is minimized. The ROFMP improves this selection process and yields a better sparsity of the solution.

We mention theoretical properties (like a convergence theorem) and demonstrate some numerical results for practical problems.

References

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Inverse problem of multiple inclusions detection in Piezoelectric Structures using XFEM and Level sets

Srivilliputtur Subbiah Nanthakumar,
Tom Lahmer, Timon Rabczuk

An algorithm to solve the inverse problem of detecting inclusion interfaces in a piezoelectric structure is proposed. The inverse problem is solved iteratively in which the Extended Finite Element Method (XFEM) is used for the analysis of the structure in each iteration. The combination of shape derivatives and level set method used in structural optimization is employed to solve the inverse problem. The formulation is presented for two and three dimensional structures and inclusions made of different materials are detected by using multiple level sets. The results obtained prove that the iterative procedure proposed can determine the location and the approximate shape of material subdomains in the piezoelectric structure by using boundary displacement and electric potential measurements. We show numerically that the solutions of the inverse problem obtained are robust with respect to errors in the data.

Regularized joint inversion of EEG and MEG data by a best basis algorithm

Sarah Orzlowski

Brain activities and conversations in the cerebrum can be described via electric signals, i. e. neuronal currents, which induce an electric potential on the scalp measured by electroencephalography *(EEG)* and a magnetic field outside the head measured via magnetoencephalography *(MEG)*. The reconstruction of the neuronal signals, the neuronal currents, from these sets of data is an ill-posed inverse problem, since the radial component of the current cannot be reconstructed. Hence, we decompose the current into suitable functions and analyze the null spaces of the associated operators.

It is now our aim to reconstruct and localize the electric currents in the brain by means of the decomposition and the novel *regularized functional matching pursuit algorithm (RFMP)*, which has been used so far for tomographic inverse problems in geophysics. This algorithm needs an appropriate set of trial functions, the so-called *dictionary*, which we construct for the particular problem, a regularization term, and a regularization parameter, which has to be chosen wisely in order to obtain a suitable solution. With this algorithm, separate inversions of the EEG and MEG data are conducted primarily, followed by a joint inversion of both data sets. Some numerical results for a test case are demonstrated.

Aggregation of Regularized Approximations

Sergei V. Pereverzyev

Recent satellite missions monitoring the Earth's gravity or magnetic fields supply a large amount of data with a fairly good global coverage. In order to obtain high-resolution gravitational or geomagnetic models it become necessary to combine these data. For example, different types of observations such as Satellite-to-Satellite Tracking (SST) or Gradiometer Measurements (SGG) enter into the determination of gravity field, leading to several observation equations, which have the same solution, namely the gravitational potential at the Earth's surface. A naive way would be to estimate this potential with the use of only one of the observation equations. At the same time, the goal is to benefit from the diversity of observation models. In the talk we are going to discuss the possibility to achieve this goal by aggregating regularized solutions of all observation equations. The idea of aggregation was originally proposed in the context of statistical regression analysis that corresponds to the case of observation equations with identity operators. Here we develop this idea in the context of inverse problems.

The presentation is based on the results of joint research with Chen Jieyang and Pavlo Tkachenko, both from RICAM. The research is partially supported by the Austrian Science Foundation FWF.

Multi-penalty regularization for detecting relevant variables

Sergiy Pereverzyev Jr.,
Kateřina Hlaváčková-Schindler, Valeriya Naumova

In this talk we present a new method for detecting relevant variables from a priori given high-dimensional data under the assumption that input-output dependence is described by a nonlinear function depending on a few variables. The method is based on the inspection of the behavior of discrepancies of a multi-penalty regularization with a component-wise penalization for small and large values of regularization parameters. We provide the justification of the proposed method under a certain condition on sampling operators. The effectiveness of the method is demonstrated in the example with synthetic data and in the reconstruction of gene regulatory networks. In the latter example, the obtained results provide a clear evidence of the competitiveness of the proposed method.

Identification of Chemotaxis Models with Volume Filling

Jan-Frederik Pietschmann,
Herbert Egger, Matthias Schlottbom

Chemotaxis refers to the directed movement of cells in response to a chemical signal called chemoattractant. A crucial point in the mathematical modelling of chemotatic processes is the correct description of the chemotactic sensitivity f and of the production rate g of the chemoattractant. In this talk, we investigate the identification of these non-linear parameters in a chemotaxis model with volume filling given by

$$\begin{aligned} \partial_t \rho &= \nabla \cdot (\nabla \rho - f(\rho) \nabla c) \qquad \text{in } \Omega \times (0,T), \\ 0 &= \Delta c - c + g(\rho) \quad \text{in } \Omega \ \text{ for a.e. } t \in (0,T), \end{aligned}$$

with $\Omega \subset \mathbb{R}^2$ and complemented by appropriate initial and boundary conditions. We give an affirmative answer to the question of unique identifiability of either f or g given distributed measurements of the cell density $\rho = \rho(x,t)$. Furthermore, we discuss the numerical realization of Tikhonov regularization for its stable solution. Our theoretical findings are supported by numerical tests and conclude by presenting some open problems.

Recovering shape of 2D pipe with corrosion and attenuation coefficient with limited data

Zenith Purisha, Samuli Siltanen

The inverse problem of reconstructing 2D pipe with corrosion using Bayesian Inversion and Non-Uniform Rational B-Splines (NURBS) is considered. This method is implemented to assess the shape of corrosion and the attenuation coefficient of the inside material out of limited measurement. Instead of points of curve, control points in NURBS become our parameters of interest. Accordingly, Markov Chain Monte Carlo is recommended to manage the emersion of non-linearity. The outside boundary of the pipeline and the attenuation coefficient of steel are as a priori information. The potential drawback of the proposed method is heavy computation.

Keywords

tomographic; corrosion; NURBS; Bayesian inversion; MCMC

Solution-functional and data-functional regularization strategies for determining the laser beam quality parameters

Teresa Regińska

The laser beam is an electromagnetic field so complicated that its analytical description is practically impossible. Especially, semiconductor lasers have the large divergence of the beam, the multimode structure of the field and often the astigmatism. An experimental determination of the whole radiation field is also impossible. Practically, we are able to measure the electromagnetic field only on some subsets of physical space (e.g. on some surfaces).

Therefore, numerical solving the ill-posed problem of reconstruction of the radiation field from experimental data given on a part of domain boundary is very important. The presentation will concern the simplified mathematical model for collimated laser beam that is described by Cauchy problem for the Helmholtz equation on bounded domain, especially on cuboid with measurements accessible on the one its side only. The case of infinite strip was already considered in [1, 2] where numerical analysis of spectral type regularization for the problem was presented. The case of rectangular domain was considered in [3] where using a certain series representation of solution, we formulate a finite dimensional spectral type regularization method. The obtained error bound depends on the regularization parameter, measurement error and, additionally, on an *a priori* bound for the solution traces on the part of boundary, where no measurements are performed.

Now, we are interested in determining certain laser beam quality parameters that can be described by corresponding functionals. One strategy consists in direct applying the regularized radiation field in the functionals (solution-functional strategy). Weaknesses of this approach will be explained. Another strategy (data-functional one) based on additional measurements and *a priori* assumptions will be presented. In this approach quality parameters are estimated directly from measurement data.

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An entropic Landweber-type method for linear ill-posed problems

Elena Resmerita

Iterative methods which approximate non-negative solutions of linear ill-posed problems are of interest in numerous applications. This talk presents such a procedure which has a relatively simple closed formula.

Atmospheric tomography for ELT Adaptive Optics

Daniela Saxenhuber, Ronny Ramlau

The problem of atmospheric tomography arises in ground-based telescope imaging with Adaptive Optics (AO), where one aims to physically correct atmospheric turbulences via deformable mirrors in real-time, i.e. at around 500 Hertz. The calculation of the optimal mirror deformations from wavefront sensor measurements is an ill-posed inverse problem. Furthermore, there is a strong increase in the computational load for atmospheric reconstruction for the upcoming generation of Extremely Large Telescopes (ELT). Thus, in order to achieve a reconstruction within the required time frame, numerically cheap iterative solvers that yield a superior reconstruction quality are needed. In this talk, we propose to use a three step approach with a Gradient-based method for the atmospheric tomography problem. We will present numerical results of our algorithm in the context of the European Extremely Large Telescope (E-ELT) that is currently under construction.

Identification of nonlinear heat conduction laws in heat transfer problems

Matthias Schlottbom,
Herbert Egger, Jan-Frederik Pietschmann

We consider the identification of nonlinear heat conduction laws in stationary heat transfer problems. Only a single additional measurement of the temperature on a curve on the boundary is required to determine the unknown parameter function on the range of observed temperatures. We first present a new proof of Cannon's uniqueness result. For a stable solution we then utilize a reformulation of the inverse problem as a linear ill-posed operator equation with perturbed data and operators. We can then prove convergence and convergence rates of the regularized reconstructions under mild assumptions on the exact parameter. These are, in fact, already needed for the analysis of the forward problem and no additional source conditions are required. Numerical tests are presented to illustrate the theoretical statements. We will indicate some generalizations of our results to parabolic problems, and to problems involving lower order terms.

Well-Posed Bayesian Geometric Inverse Problems

Andrew Stuart,
Marco Iglesias, Kui Lin, Yulong Lu

There are numerous inverse problems where geometric characteristics, such as interfaces, are key unknown features of the overall inversion. Applications include the determination of layers and faults within subsurface formations, and the detection of unhealthy tissue in medical imaging. We discuss a theoretical and computational Bayesian framework relevant to the characterization of such inverse problems.

We start with models in which the geometry is defined via a finite number of parameters, utilizing descriptions which are sufficiently general to include layered media, faults and channels, and to allow for spatial variability within the different components of the field [1]. We then proceed to demonstrate how to use level set formulations of the Bayesian inverse problem, allowing for more complex geometric interfaces which are described by an infinite set of parameters [2].

For a groundwater flow inverse problem, in which hydraulic head measurements are used to condition the prior information on the permeability, we show that the Bayesian formulation gives rise to a well-posed posterior distribution, suitable for numerical interrogation. We then describe numerical experiments which explore the posterior distribution, using state-of-the art MCMC methods.

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Multi-parameter regularization of ill-posed spherical pseudo-differential equations in C-space

Pavlo Tkachenko

In this talk, a two-step regularization method is considered to solve an ill-posed spherical pseudo-differential equation. For the first step of regularization we use the approximation of continuous functions by means of spherical polynomials minimizing a functional with a penalty term in reproducing kernel Hilbert space. The second step is the regularized collocation method. The error bound for the constructed multi-parameter regularization is established in C-norm. We discuss an a posteriori parameters choice rule and present some numerical experiments to confirm the compensatory property of our method.The present research was performed in joint cooperation with Dr. Hui Cao (China), Prof. Dr. Ian H. Sloan (Australia), and Prof. Dr. Sergei Pereverzyev (Austria)

Statistical Inverse Problems in fluorescence microscopy

Frank Werner

In this talk we discuss several inverse problems arrising in the application of fluorescence microscopy. In the most clasical setup, one tries to reconstruct a unknown fluorophore marker density u from photon counts of the density $g_1 = k * u$. Here k is a convolution kernel (called point-spread function) and g denotes exact data. Unfortunately, one is not only interested in approximations of u, but one wants to determine a so-called molecular map. This is knowing the number n of (active) molecules or markers on each grid point x. If all markers at x share the same brightness $p\,(x)$, then $u = p \cdot n$. Obviously, p and n cannot be identified given only g_1. This can be overcome by exploiting the effect of antibunching, which yields additional photon counts of a density g_2 which is depends quadratically on p and n. We will discuss problems in determining n from g_1 and g_2, including non-integer reconstuctions. Additionally we will focus on statistical tests for the hypothesis $n\,(x) = 0$ against a meaningful alternative.

Adaptive sensitivity-based regularization for Newton-type inversion in Electrical Impedance Tomography

Robert Winkler, Andreas Rieder

Electrical impedance tomography is the inverse problem of determining the electrical conductivity distribution of an object from measurements at electrodes on its boundary. Due to the ill-posedness of this problem, small measurement and modelling errors can have a huge impact on the recovered image. Iterative inversion algorithms thus incorporate regularization techniques, e.g. smoothness priors, to recover the underlying conductivity.

Using information of the Fréchet derivative of the forward operator, which is available at no additional computational cost in Newton-type algorithms, we accurately model the impact of conductivity changes to boundary measurements. This sensitivity information can be incorporated into an adaptive discretization scheme for the inverse problem. Each conductivity coefficient in the resulting discretization is equally sensitive to measured boundary data, that is, to conductivity information and noise contained therein. This allows for more accurate regularization and increases the robustness of the inversion scheme.

Using an inexact Newton-type method with a discrepancy principle, a robust choice of regularization parameters is reduced to the estimation of the noise level of the measured data. We demonstrate the performance of this scheme with simulated and measured data.

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Inverse problems of moving sources in wave equation

Masahiro Yamamoto

We consider a wave equation with source term moving in time. We discuss the uniqueness and stability in determining the source intensity and velocity by over-determining boundary data. Also we discuss the case of zero velocity and propose a numerical method.

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