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The comparative statics of effective demand

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Abstract

Keynes introduces the term 'effective demand' in chapter 3 of the *General Theory* as designating the point of intersection of two functions: the 'aggregate demand function' (D) and the 'aggregate supply function' (Z). For the first time in the literature, I here specify exact functional forms for the D and Z functions and run numerical simulations which allow to study the comparative statics of the model in the face of various 'shocks'. The demonstration of how the D/Z model actually works will hopefully prove useful for future students of the economics of Keynes.

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1. Introduction

Effective demand is a nasty subject. So much ink has been spilled over it already and still you cannot be sure what your colleague means when she uses the term. Most people probably think of effective demand as suggesting that aggregate demand determines aggregate supply as in Samuelson's model of the 'Keynesian Cross'. The 'Keynesian Cross' does not feature in the *General Theory* (Keynes 1936), however. In that book, Keynes introduces the term 'effective demand' in chapter 3 as designating the point of intersection of two functions: the 'aggregate demand function' (D) and the 'aggregate supply function' (Z). The mainstream interpretation of Keynes has largely ignored the D/Z model of chapter 3, concentrating on Hicks's (1937) IS/LM model instead. It is the founding co-editors of the *Journal of Post Keynesian Economics*, Sidney Weintraub and Paul Davidson, who share the merit of having rescued Keynes's D/Z analysis from oblivion.[1] Still, however, controversy persists over the correct interpretation of the chapter 3 model.

My aim in this paper is *not* to recapitulate earlier debates over the correct interpretation of Keynes's D/Z model.[2] Rather, my aim is to demonstrate that the interpretation I have advocated in these debates allows for writing down explicit equations for the Z and D curves and setting the model in dynamic motion. It will thereby be possible to simulate equilibria of the whole model and to conduct comparative static exercises in the face of various 'shocks'. This has never been done in the earlier literature,[3] although it is very useful because it proves that the D/Z model actually works.[4] The classroom, where students are taught the economics of Keynes, is thus the place the present paper ultimately targets.[5]

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[1] See Weintraub (1958), Davidson and Smolensky (1964) and Davidson (1978, 1994, 2002). King (1994) scrutinizes the early stages of non-Post Keynesian aggregate supply and demand analysis. "To conclude that there was some confusion about aggregate supply and demand analysis in the early 1950s would be a grotesque understatement", he sums up (p. 14).

[2] Some recent contributions to this debate include Hayes (2007a, 2007b, 2008), Hartwig and Brady (2008), Allain (2009), Palacio-Vera (2009), Ambrosi (2011), Hartwig (2011), Allain et al. (2013), Allain (2013), Hartwig (2013) and Hayes (2013).

[3] There exists a literature (including Chick 1983, Galbraith and Darity 2005 and Lawlor 2008) which has formalized Keynes's D/Z model. But never before have exact functional forms for the D and Z functions been specified and shocks to model equilibria been simulated.

[4] This has been questioned by Lavoie (2003), for instance.

[5] Hagemann (2010) recollects that the D/Z model – which he calls the 'fish diagram' – had a very limited appeal for students in Paul Davidson's course 'Money and Banking' at the Graduate Faculty of the New School for

The paper is organized as follows. The next section summarizes the interpretation of Keynes's chapter 3 model underlying the simulations, and section 3 presents the latter. Section 4 concludes.

2. Keynes's model of effective demand

In chapter 3 of the *General Theory*, Keynes develops the principle of effective demand in the context of a thought experiment by entrepreneurs, who aim at maximizing profit. To understand the principle, it is important to visualize the economic process as a sequence of production periods. Entrepreneurs plan for a certain period of the future and are bound by their decisions until the end of the period. The principle of effective demand is what guides their planning. To simplify the exposition, let us assume that the individual plans can be aggregated straightforwardly and that the planning period is the same for all entrepreneurs.

Keynes models the entrepreneurs' planning task in terms of two functions, the 'aggregate supply function' (Z) and the 'aggregate demand function' (D). Z is "the aggregate supply price of the output from employing N men" (Keynes 1936: 25). The aggregate supply price is defined by Keynes as "the expectation of proceeds which will just make it worth the while of the entrepreneur to give that employment" (Keynes 1936: 24).

Keynes defines *Z* as the product of an aggregate price and output component. The latter, the 'output of N men', we can identify as net value added[6] (which is dependent on employment). Keynes uses the symbol 'O' for aggregate output in chapter 21 of the *General Theory,* but I prefer the more familiar expression Y(N) for the aggregate production function.[7]

Social Research in 1986 when Hagemann was a Visiting Professor there. To improve the didactics of that model is thus an important task.

[6] Not gross value added because Keynes subtracts what he calls 'user cost' – the sum of intermediate consumption and depreciation allowances – from gross output in the aggregate (see Keynes 1936: 23-24). Curiously, on p. 299 of the *General Theory*, Keynes writes that effective demand corresponds to "gross, not net, income" which indicates that he was willing to include the depreciation part of the user cost in D and Z. For the purpose of this paper it makes no difference whether Y(N) stands for gross or net value added.

[7] Some (post-Keynesian) economists, e.g. Shaik (1974), Felipe and Fisher (2003) and Felipe and McCombie (2006), have criticized aggregate production functions, mainly for including 'K' as a measure of physical capital despite known measurement problems and the Cambridge (UK) 'capital critique'. Keynes, who sympathized with "the pre-classical doctrine that everything is *produced* by *labour*" (Keynes 1936: 213, emphasis in the original) and therefore regarded "labour […] as the sole factor of production" (ibid.: 213-214), did not include physical capital in his production function. The latter is therefore not much affected by the above-mentioned critique. Note that the question whether it is admissible to interpret Keynes's chapter 3 model in terms of an

The price level implicit in Keynes's aggregate supply function, P^s, must have the property that the proceeds it generates "will just make it worth the while of the entrepreneurs to give that employment" – in other words, P^s must be the profit-maximizing price level. With respect to the micro-foundations of aggregate supply, the *General Theory* does not part company with the (neo)classical approach. Therefore, Keynes (1973: 24-25) takes it for granted that the "entrepreneurs will endeavor to fix the amount of employment at the level which they expect to maximize the excess of proceeds over the factor cost".[8]

The mathematical approach to find out that level is standard. Simply differentiate the profit function with respect to employment to obtain the first-order condition. From this, the profit-maximizing supply-price level P^s can be derived (see equations 1 and 2).[9]

$$\Pi = P^s \cdot Y(N) - w \cdot N \tag{1}$$

$$\frac{d\Pi}{dN} \overset{!}{=} 0 \Rightarrow P^s \cdot \frac{dY}{dN} - w = 0 \Rightarrow P^s = w \cdot \frac{dN}{dY} \tag{2}$$

Z, being the mathematical product of the output and the supply price levels, is thus given by (3):

$$Z = P^s \cdot Y(N) = w \cdot \frac{dN}{dY} \cdot Y(N) = \frac{w}{Y'(N)} Y(N) \tag{3}$$

Under decreasing marginal returns to labor, P^s grows progressively while Y(N) grows with diminishing returns. Altogether, Z might be a linear function of N. At least, this seems to be hinted at in a somewhat opaque footnote on pp. 55-56 of the *General Theory* in which Keynes suggests two arguably inconsistent things: first, that Z in wage units (= Z_w) is linear with a slope of 1 and second, that the slope of Z_w is given by the reciprocal of the money wage. Ambrosi (2011) has recently suggested that the second proposition could be made sense of if the word 'share' was added at the very end. In other words, Ambrosi argues that the slope of Z_w was given by the inverse of the money wage *share*. This can be proven in a rigorous way as follows.

Z in wage units equals Z divided by *w*. From equation (3) it follows:

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aggregate production function was at issue in the debate between Hayes (2007b, 2008) and Hartwig and Brady (2008).

[8] Palley (1997) and others have pointed out that Keynes's acceptance of the 'first classical postulate' (Keynes, 1973: 17-18) implies the adoption of the neo-classical supply-side assumptions of free competition, price-taking, profit-maximization, and decreasing marginal returns to labor. I honor these assumptions in what follows.

[9] With Π = aggregate profit, P^s = aggregate supply price level, Y = net value added, N = employment, w = wage unit (= average nominal wage rate, see Keynes 1936: 41).

$$Z = \frac{w}{Y'(N)} \cdot Y(N) \Leftrightarrow Z_w = \frac{Y(N)}{Y'(N)} \tag{4}$$

Remembering that Z equals $P^s \cdot Y(N)$, we know that Z_w equals $(P^s / w) \cdot Y(N)$ or $P_w^s \cdot Y(N)$. Then we can write:

$$\frac{d(P_w^s \cdot Y(N))}{dN} = \frac{d(P_w^s \cdot Y(N))}{dN} \cdot \frac{1}{dY(N)/dN} \cdot Y'(N) = \frac{d(P_w^s \cdot Y(N))}{dY(N)} \cdot Y'(N) \tag{5}$$

Knowing that $Y'(N) = w / P^s = 1 / P_w^s$ because points on Z are profit-maximizing, we can rewrite (5) as:

$$\frac{d(P_w^s \cdot Y(N))}{dY(N)} \cdot Y'(N) = \frac{d(P_w^s \cdot Y(N))}{dY(N)} \cdot \frac{1}{P_w^s} \tag{6}$$

Keynes calls $P_w^s \cdot Y(N)$ 'D_w' in chapter 20 of the *General Theory*. So we can rewrite (6) as:

$$\frac{d(P_w^s \cdot Y(N))}{dY(N)} \cdot \frac{1}{P_w^s} = \frac{dD_w}{dY(N)} \cdot \frac{Y(N)}{D_w} \tag{7}$$

On the right-hand side of equation (7) we have the inverse of what Keynes (1936: 282-283) defines as 'the elasticity of output or production'. This is therefore the most general expression for the slope of the 'aggregate supply curve' (in wage units): it equals the inverse of the output elasticity. Under conventional (neo)classical supply-side assumptions, which Keynes accepted in the *General Theory*, the inverse of the output elasticity equals the inverse of the wage share.[10] So Ambrosi's conjecture on the last sentence of the footnote on pp. 55-56 of the *General Theory* is correct; and the result can be derived in a straightforward way.

Now let us turn to the 'aggregate demand function' D. According to Keynes, it gives "the proceeds which entrepreneurs expect to receive from the employment of N men" (Keynes 1936: 25). In a diagram with employment as abscissa and expected proceeds as ordinate, which Keynes describes verbally on page 25 of the *General Theory*, D lies above Z for small N. At a certain point – corresponding to a certain level of employment N – however, D and Z intersect. Keynes calls this point of intersection 'effective demand' and states that "it is at this point that the entrepreneurs' expectation of profits will be maximised" (Keynes 1936: 25).

The interpretation of this passage of the *General Theory* is straightforward if we remember that Keynes adopted the (neo)classical micro-assumptions of profit-maximization and price-

[10] Davidson and Smolensky (1964: 125, 134-135) discuss the connection between the wage share and the slope of Z and show that Z is linear for the production function $Y = N^{\alpha}$ which I assume below.

taking.[11] Because entrepreneurs cannot hope to dictate prices neither in their individual markets nor at the aggregate level they use the calculus of equations (1) and (2) to find out which price level would maximize profits. P^s, the price level implicit in *Z*, is in a way purely hypothetical. *If*, for a certain N_1, the entrepreneurs expected the price level given by (2) to rule in the market they would employ N_1 men because they knew that profits would thereby be maximized. But which price level do they *really* expect? This question is not answered by the supply function at all but by the demand function. The price level implicit in D, which we can call the demand price level P^d, is the price level the entrepreneurs really expect to rule in the market. Hence Keynes writes "let D be the proceeds which entrepreneurs expect to receive from the employment of N men". If, for a certain N, $P^d > P^s$, "there will be an incentive to entrepreneurs to increase employment beyond N and, if necessary, to raise costs by competing with one another for the factors of production, up to the value of N for which Z has become equal to D" (Keynes 1936: 25).

3. Simulating the D/Z model

This section goes beyond the existing literature in performing numerical simulations with the D/Z model. To do so, it is first of all necessary to specify a production function. As was mentioned above, Keynes regarded labor as the sole factor of production and he assumed decreasing marginal returns to labor throughout the *General Theory*.[12] I therefore choose the production function $Y = N^{0.7}$. The first line of Table 1 lists the assumed production function as well as the functions Z and D. The nominal wage rate, which enters Z (see equation 3 above), is normalized to 1. For D, I assume that the entrepreneurs expect a demand price level of 5.

<Insert Table 1 around here>

[11] Keynes's notion of price-taking departs from the strict microeconomic theory of the small firm operating under perfect competition. That theory would not allow for entrepreneurs forming ex ante expectations about demand. Keynes – who was concerned with the real world – did not have such firms in mind. In his theory, firms are not 'atomistic', but also not powerful enough to dictate the price. They have to form expectations about the price for their products the market will accept and about the market share that might be attributable to them (see Chick 1992).

[12] In his essay "Relative Movements of Real Wages and Output", Keynes (1939) relaxed the assumption of decreasing marginal returns to labor in the short period. In a companion paper (Hartwig 2016), I discuss possible reasons that might have prompted Keynes to relax his core assumption from the *General Theory* and analyze the consequences of assuming non-decreasing marginal returns to labor for the model of effective demand.

Figure 1 shows the D and Z curves for the first set of assumptions. The slope of Z equals the inverse of the output elasticity, which corroborates the theoretical derivation above.[13] Employment on the abscissa runs from 1 to 70. D and Z intersect at an employment level between 65 and 66. At this employment level, the marginal product of labor (MPL) lies between 0.200 and 0.199, which is equal to the real wage for the assumed values w = 1, $P^d = 5$.

<Insert Figure 1 around here>

We can also add investment demand.[14] In chapter 3 of the *General Theory*, Keynes distinguishes between two components of D, which he calls D_1 and D_2. D_1 designates expected consumption demand and is, according to Keynes (1936: 28-29), a function of employment: $\chi(N)$. Although he does not say it directly, from what he writes on page 30 of the *General Theory* it is clear that Keynes regarded expected investment demand (D_2) *not* to be a function of employment (see also Chick 1983: 67). This means that if we draw D_2 in the PY/N space, it should be a horizontal line – with the concave D_1 curve set on top of it.[15] In the first simulation it was implicitly assumed that the entrepreneurs expect zero investment. This implied zero savings and a propensity to consume of 1.

If we now assume positive investment (expectations) we must also relax the assumption that the propensity to consume equals 1. We can use the model to calculate which propensity to consume is consistent with a certain expected level of investment. For example, if the expected level of investment equals 10, the D curve becomes

$$D = c \cdot P^d \cdot Y(N) + 10 \tag{8}$$

If we still assume a demand price level of 5, we know that the real wage still equals 0.2. A marginal product of labor consistent with this real wage level will still be associated with an employment level around 65. So we are looking for a value of *c* that generates a value for D equal to 92.86 (the value of Z for an employment level of 65, see Figure 1). We can calculate *c* from equation 9:

[13] I showed in section 2 that the slope of Z in wage-units (Z_w) is equal to the inverse of the output elasticity, i.e. equal to $1/\alpha$. So the slope of Z is equal to w/α. Because of the normalization of the money wage rate to 1 we have the special case that the slope of Z is the same as that of Z_w.

[14] The D/Z model refers to the Marshallian short period for which the impact of investment on the capital stock is disregarded.

[15] It is the entrepreneurs in the consumption-goods sector who have to form expectations about the level of investment spending in order to calculate how much demand will be forthcoming to them through the multiplier mechanism (see Hartwig 2004).

$$92.86 = c \cdot 5 \cdot 65^{0.7} + 10 \Leftrightarrow c = 0.89 \quad (9)$$

Savings ($0.11 \cdot 5 \cdot 65^{0.7}$) are equal to the investment of 10 (apart from rounding errors).

We can also assume negative investment, for instance D_2 = -5. Keeping all other parameters the same as in the previous simulation now yields two equilibria at employment levels N_1 = 2.63 and N_2 = 30.85. N_1 is an unstable equilibrium, however, since Keynes postulates that entrepreneurs want to produce more and increase employment whenever $P^d > P^s$. This could be formalized in a differential equation $\dot{N} = \theta \cdot (P^d - P^s)$ with $\theta > 0$, which assures that N moves in the 'correct' direction to make N_1 an unstable equilibrium and N_2 a stable one.

Now let us return to the model without investment and assume a 'supply shock'. For instance, due to a positive shock to productivity, let the output elasticity rise from 0.7 to 0.75.[16] The second line of Table 1 lists the production function as well as Z and D for the second set of assumptions, and Figure 2 shows the result of the simulation. The Z curve moves down. Its slope is now the inverse of 0.75, so it is flatter than before. The D curve moves up because every unit of employment now produces more real income and expected demand. The new point of intersection lies at an employment level between 197 and 198. Again, the marginal product of labor is close to the real wage of 0.2.

<Insert Figure 2 around here>

We can also simulate a 'demand shock' as a rise or decrease in the demand price level. Keynes's acceptance of the 'first classical postulate' according to which (i) the real wage is equal to the marginal product of labor (see Keynes 1936: 5) and (ii) the marginal product of labor is decreasing so that output and the real wage move in opposite directions in the short period (see Keynes 1936: 17-18) implies that entrepreneurs expand output (and employment) only if they expect that the market will accept the price increase necessary to cover the rise in marginal cost (due to decreasing marginal returns). In other words – absent positive supply shocks – entrepreneurs must be able to expect a higher demand price level in order to expand output. For this reason, Keynes wrote in an open letter to President Roosevelt:

> "Rising prices are to be welcomed because they are usually a symptom of rising output and employment. When more purchasing power is spent, one expects rising output at rising prices. Since there cannot be rising output without rising prices, it is essential to

[16] An improved organization of the production process might cause such a shock.

> insure that the recovery shall not be held back by the insufficiency of the supply of money to support the increased monetary turnover" (Keynes 1933: 33).[17]

Figure 3 shows the consequences of raising the demand price level from 5 to 6 (without specifying yet which improvement in demand conditions prompted entrepreneurs to revise their demand price expectations upward). Table 1 (row 3) gives the details for the third simulation. The D curve moves to the top, employment and output rise, and the marginal product of labor drops to the value of the new real wage rate (of 1/6).

<Insert Figure 3 around here>

Now assume that a positive shock to investment expectations is the reason for the improvement in demand conditions. This shock can raise employment only if it is associated with a higher expected demand price level (or a lower nominal wage rate). If it is not, something inconsistent occurs, as is shown in Figure 4. The simulation assumes a jump in expected investment from 10 to 20 without a change in the demand price level or the nominal wage rate. The figure shows that output and employment rise. If output and employment are higher than in simulation 1, the marginal product of labor must be lower. The table accompanying Figure 4 shows that the MPL drops from 0.2 to around 0.186. Still, the real wage is unchanged at 1/5. So there is a discrepancy between the marginal product of labor and the real wage, which, according to Keynes, must be cured by a price reaction. Hence the market price – as opposed to the 'demand price', which is an *ex ante* expectation of the entrepreneurs – will rise to a value of around 5.38 in order to equalize the marginal product and the real wage.

<Insert Figure 4 around here>

If the entrepreneurs fully anticipate the situation, they know that their output expectation of around 22 is inconsistent with their demand price expectation of 5. So they will probably plan with a higher demand price in the first place. Of course, this would move the D curve further upward, leading to an equilibrium with an even lower marginal product of labor. On the other hand, the entrepreneurs can expect that the workers will resist ever lower real wages and will start asking for higher nominal wages. This moves the Z curve inwards and can lead to an equilibrium closer to the origin.

[17] Note that Keynes does not indicate that rising prices raise output and employment because they lower the real wage rate. They are a 'symptom', not a cause of rising output and employment.

The interaction between wages and demand prices distinguishes Keynes's model of effective demand from the neoclassical labor market model. In the latter, the elasticity of labor demand with respect to the wage rate is strictly negative because an increase in the nominal wage rate leaves the price level unaffected so that the real wage rises. The economy moves up the negatively sloped demand curve for labor, which means that employment goes down. In Keynes's model, however, there is no such demand curve for labor. Entrepreneurs anticipate that if money wages rise there will be more purchasing power in the economy; so they can expect that the market will accept a higher price level. In other words, the demand price level – an *expected* value which nevertheless governs entrepreneurs' production decisions – rises and falls with the nominal wage rate. For this reason, no clear-cut statements about the elasticity of labor demand with respect to the nominal wage rate are possible: all depends on the elasticity of the demand price level with respect to the nominal wage rate. If this elasticity is greater than one, an (expected) rise in the nominal wage rate can raise labor demand instead of lowering it. This is one upshot of chapter 19 of the *General Theory* on 'Changes in Money-Wages'.

Another upshot of chapter 19 is that changes in nominal ('money') wages – or the expectation that nominal wages will change – not only have an impact on the 'short-term' expectations concerning the demand price level, but also on the 'long-term' expectations concerning investment. For Keynes (1936: 46), 'short-term' expectation are concerned with "the price which a manufacturer can expect to get for his 'finished' output at the time when he commits himself to starting the process which will produce it". He observes that "in practice the process of revision of short-term expectation is a gradual and continuous one, carried on largely in the light of realised results" (Keynes 1936: 50), a remark that suggests the adoption of an adaptive approach to the revision of short term expectations. By contrast, "it is of the nature of long-term expectations that they cannot be checked at short intervals in the light of realized results. Moreover, [...] they are liable to sudden revision" (Keynes 1936: 51). That long-term expectations are viewed by Keynes as largely disconnected from on-going realizations and that they are also liable to sudden exogenous shocks is without doubt an important part of his story. In chapter 19 of the *General Theory*, however, Keynes also suggests that long-term expectations might be determined endogenously, resulting from the extrapolation into the future of observed changes in money wages:

> "If the reduction of money-wages is expected to be *a reduction relatively to money wages in the future*, the change will be favourable to investment, because […] it will increase the marginal efficiency of capital […]. If, on the other hand, the reduction leads to the expectation [...] of a further wage-reduction in prospect, it will have precisely the

> opposite effect. For it will diminish the marginal efficiency of capital and will lead to the postponement both of investment and of consumption" (Keynes 1936: 263, emphasis in the original).

The elasticity of long-term expectations with respect to the nominal wage rate can either be positive or negative, depending on whether a further decline is expected or not. If the decline is expected to continue, entrepreneurs mark down their sales expectations, which impacts negatively on the marginal efficiency of capital (see chapter 11 of the *General Theory*). If, on the other hand, wages are expected to recover, the revenue expectations remain intact; and the initial wage cut improves the marginal efficiency of capital because it reduces the supply price of capital goods. Even though the elasticity of long-term expectations with respect to the nominal wage rate is hard to determine empirically because the state of long-term expectations is an unobservable variable, the reaction of investment demand to wage shocks can be observed.

The alignment of money wages and demand prices triggered by the situation depicted in Figure 4 will eventually lead to a w/P^d combination that implies a D/Z equilibrium in which the marginal product of labor is equal to that real wage. For the sake of the argument, let us assume that the adjustment process does not take place instantaneously in the minds of the entrepreneurs at the beginning of the production period, but as a trial-and-error process over a sequence of periods. More concretely, let us assume that simulation 4 describes what happens in period 1. For period 2 let us assume that the entrepreneurs raise their demand price expectation to 5.38 and their expectation for the nominal wage rate from 1 to 1.1 (see Table 1).

Figure 5 shows the results of simulation 5.[18] The curves intersect at a smaller volume of employment than in simulation 4. Therefore, the MPL rises from 0.186 (simulation 4) to 0.19. Still, however, the MPL is lower than the *ex ante* expected real wage (of 1.1/5.38), so the market price level will rise further. Over the next periods, output and employment can be expected to decline further, while the real wage keeps rising, until the point is reached where the real wage and the marginal product of labor are equal. This is a process in which real wages and output move in opposite directions. As was noted above, Keynes (1936: 17-18) regarded this to be what generally happens in the short period.

[18] In Figure 5 it is assumed that the elasticity of long-term expectations with respect to the nominal wage rate equals zero. If this elasticity is positive, the rise in the nominal wage rate by 10 percent (from 1 to 1.1) raises (expected) investment demand (above 20); and the D curve shifts further to the top. If the elasticity is negative, however, the D curve shifts downward.

<Insert Figure 5 around here>

4. Conclusion

In chapter 3 of the *General Theory*, Keynes develops a macroeconomic model in which two functions, which he calls D and Z, determine the volume of aggregate employment at their point of intersection. Keynes (1936: 25) names Z the 'aggregate supply function', D the 'aggregate demand function', and their point of intersection 'the effective demand'. Since Keynes (1936: 89) makes it clear that "(t)he ultimate object of our analysis is to discover what determines the volume of employment", chapter 3 can be assumed to contain important insights. Yet the mainstream interpretation of Keynes has largely ignored the D/Z analysis of chapter 3, concentrating on Hicks's (1937) IS/LM model instead. Even some influential post-Keynesians – like Lavoie (2003), for instance – have argued in favor of scrapping the D/Z model so that it would become easier to recast Keynes's most valuable insights in models which are more amenable to a broader audience. Yet the D/Z model has soldiered on; and it has even experienced a little 'renaissance' recently when the *Review of Political Economy* published papers from a symposium aiming at 'securing the foundations' of effective demand.

My aim in this paper has been to demonstrate that the interpretation I have advocated in that symposium as well as in earlier debates over the 'correct' interpretation of Keynes's principle of effective demand allows for writing down explicit equations for the Z and D curves and setting the model in dynamic motion in order to simulate equilibria of the whole model and to conduct comparative static exercises in the face of various 'shocks'. This demonstration of how the D/Z model actually works will hopefully prove useful for future students of the economics of Keynes.

Acknowledgments

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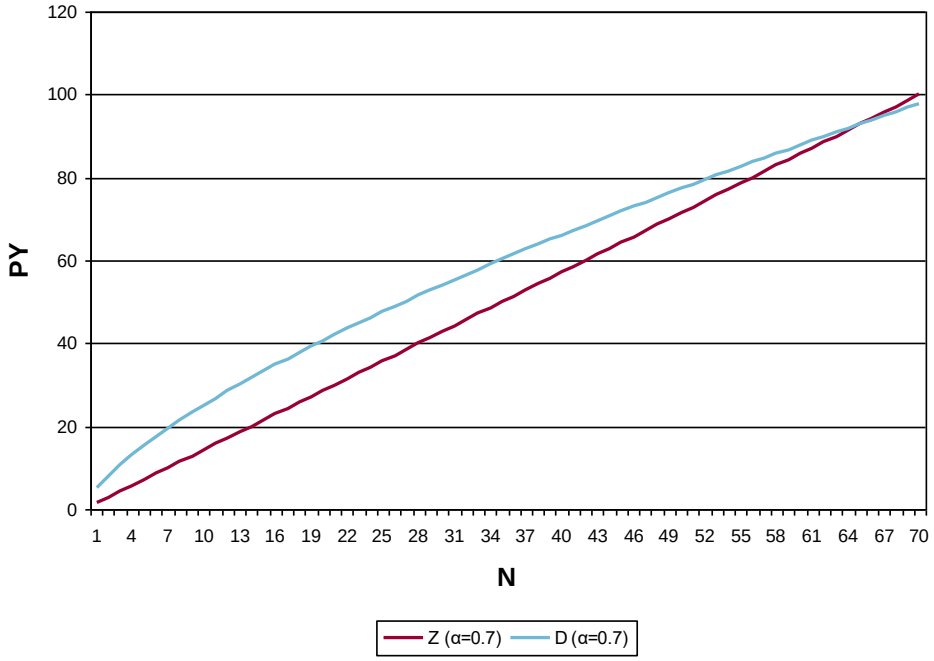
Table 1: Assumptions for numerical simulations

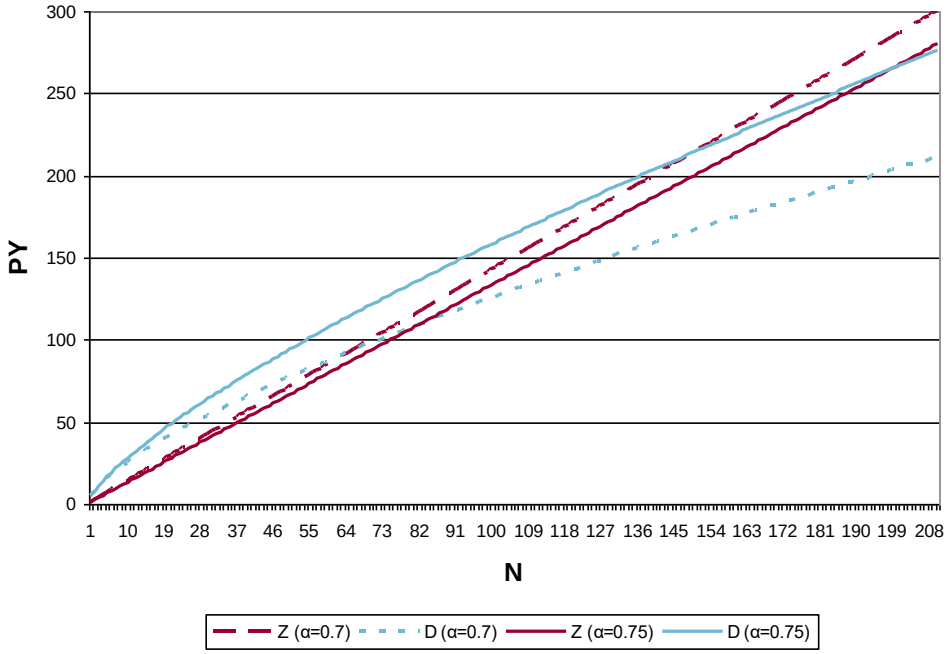
	Y(N)	**MPL**	**P^s** (=w/MPL)	**Z** (=P^s*Y(N))	**D** (=P^d*Y(N))
1	$Y=N^{0.7}$	$dY/dN=0.7*N^{-0.3}$	$P^s=1/0.7*N^{0.3}$	$Z=1/0.7*N^{0.3}*N^{0.7}$ $=1/0.7*N$	$D=5*N^{0.7}$
2	$Y=N^{0.75}$	$dY/dN=0.75*N^{-0.25}$	$P^s=1/0.75*N^{0.25}$	$Z=1/0.75*N^{0.25}*N^{0.75}$ $=1/0.75*N$	$D=5*N^{0.75}$
3	$Y=N^{0.7}$	$dY/dN=0.7*N^{-0.3}$	$P^s=1/0.7*N^{0.3}$	$Z=1/0.7*N^{0.3}*N^{0.7}$ $=1/0.7*N$	$D=6*N^{0.7}$
4	$Y=N^{0.7}$	$dY/dN=0.7*N^{-0.3}$	$P^s=1/0.7*N^{0.3}$	$Z=1/0.7*N^{0.3}*N^{0.7}$ $=1/0.7*N$	$D=0.89*5*N^{0.7}+10$ $D=0.89*5*N^{0.7}+20$
5	$Y=N^{0.7}$	$dY/dN=0.7*N^{-0.3}$	$P^s=1.1/0.7*N^{0.3}$	$Z=1.1/0.7*N^{0.3}*N^{0.7}$ $=1.1/0.7*N$	$D=0.89*5.38*N^{0.7}+2$ 0

Figure 1: Simulated D and Z functions for $Y(N)=N^{0.7}$



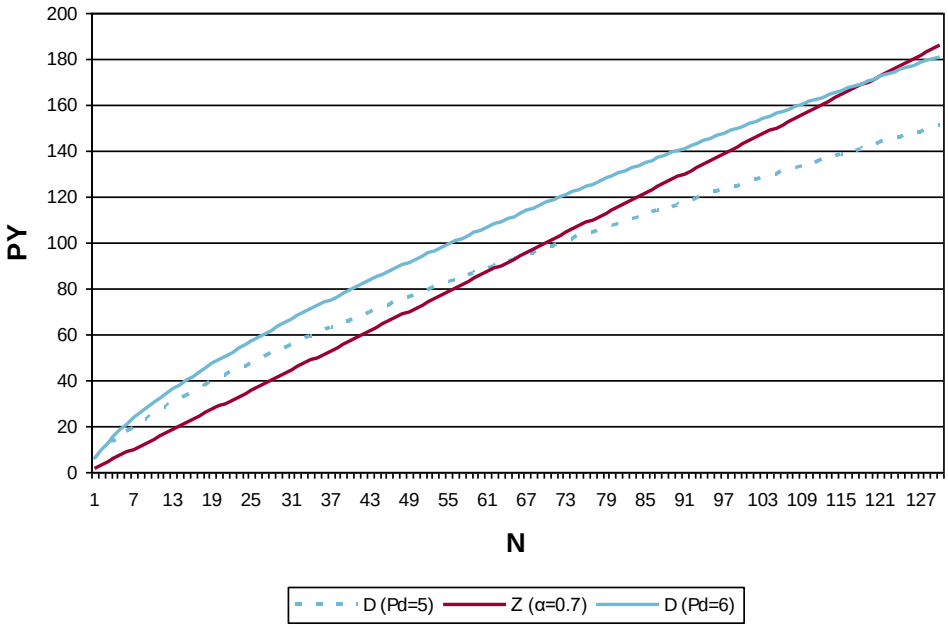
N	Y(N)	MPL	Ps (=1/MPL)	Z (α=0.7)	D (α=0.7)
61	17.7717782	0.20393844	4.90344051	87.1428571	88.858891
62	17.9752187	0.20294602	4.92741869	88.5714286	89.8760934
63	18.1776771	0.20197419	4.95112767	90	90.8883855
64	18.3791737	0.20102221	4.97457465	91.4285714	91.8958684
65	18.5797279	0.20008938	4.99776656	92.8571429	92.8986395
66	18.7793586	0.19917502	5.02071004	94.2857143	93.896793
67	18.9780839	0.19827849	5.04341146	95.7142857	94.8904194
68	19.1759213	0.19739919	5.06587692	97.1428571	95.8796066

Figure 2: Simulated D and Z functions for $Y(N)=N^{0.7}$ and $Y(N)=N^{0.75}$



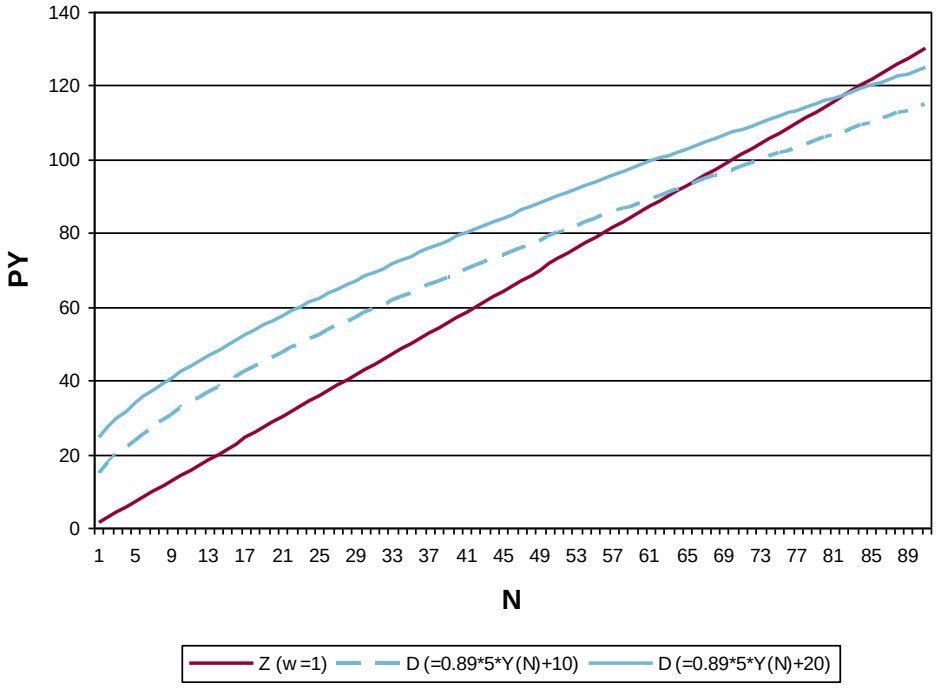
N	Y(N)	MPL	Ps (=1/MPL)	Z (α=0.75)	D (α=0.75)
193	51.7807077	0.20122037	4.96967587	257.333333	258.903539
194	51.981798	0.20096056	4.9761008	258.666667	259.90899
195	52.1826294	0.20070242	4.98250094	260	260.913147
196	52.3832034	0.20044593	4.98887652	261.333333	261.916017
197	52.5835218	0.20019107	4.99522774	262.666667	262.917609
198	52.7835861	0.19993783	5.00155483	264	263.91793
199	52.983398	0.19968617	5.007858	265.333333	264.91699
200	53.182959	0.1994361	5.01413746	266.666667	265.914795

Figure 3: Simulated D and Z functions for P^d=5 and P^d=6



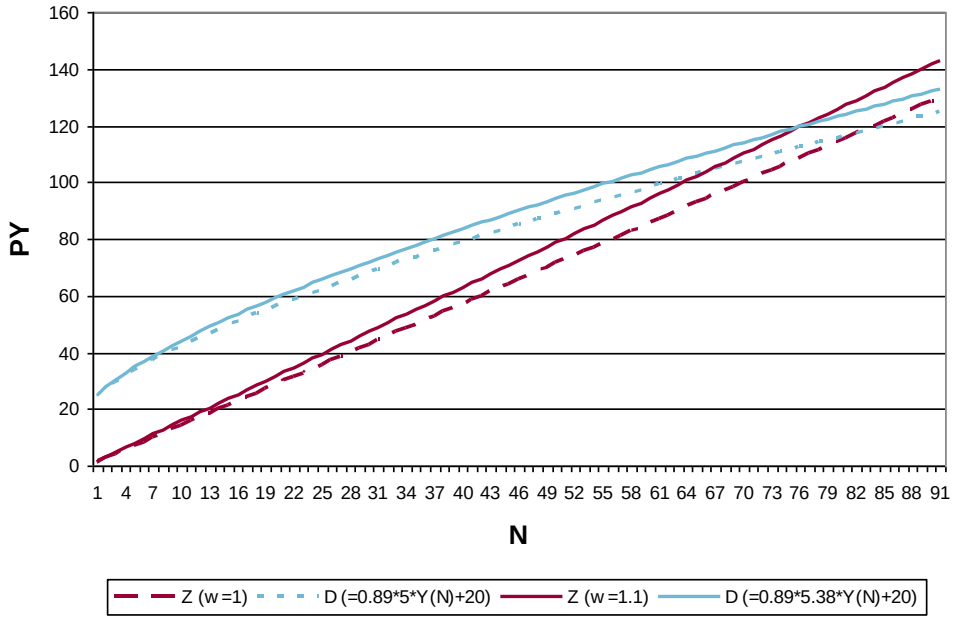
N	Y(N)	MPL	Ps (=1/MPL)	Z (α=0.7)	D (Pd=6)
115	27.7005561	0.16861208	5.93077314	164.285714	166.2033368
116	27.8689491	0.16817469	5.94619787	165.714286	167.2136946
117	28.0369071	0.16774218	5.96152979	167.142857	168.2214428
118	28.2044351	0.16731445	5.97677026	168.571429	169.2266103
119	28.3715376	0.1668914	5.99192058	170	170.2292256
120	28.5382194	0.16647295	6.00698204	171.428571	171.2293163
121	28.704485	0.166059	6.0219559	172.857143	172.2269099
122	28.8703389	0.16564949	6.03684339	174.285714	173.2220332

Figure 4: Simulated D and Z functions for D=0.89*5*Y(N)+10 and D=0.89*5*Y(N)+20



Y(N)	MPL	Ps (=1/MPL)	Z (w=1)	D (=0.89*5*Y(N)+10)	D (=0.89*5*Y(N)+20)
17.7717782	0.20393844	4.90344051	87.14285714	89.26213079	99.26213079
17.9752187	0.20294602	4.92741869	88.57142857	90.16947533	100.1694753
18.1776771	0.20197419	4.95112767	90	91.07243988	101.0724399
18.3791737	0.20102221	4.97457465	91.42857143	91.97111461	101.9711146
18.5797279	0.20008938	4.99776656	92.85714286	92.86558647	102.8655865
18.7793586	0.19917502	5.02071004	94.28571429	93.75593932	103.7559393
18.9780839	0.19827849	5.04341146	95.71428571	94.64225412	104.6422541
19.1759213	0.19739919	5.06587692	97.14285714	95.52460905	105.524609
19.3728878	0.19653654	5.08811229	98.57142857	96.40307965	106.4030796
19.5689998	0.19569	5.11012322	100	97.27773895	107.2777389
19.764273	0.19485903	5.13191512	101.4285714	98.14865758	108.1486576
19.9587228	0.19404314	5.15349322	102.8571429	99.01590391	109.0159039
20.1523641	0.19324185	5.17486254	104.2857143	99.87954409	109.8795441
20.3452113	0.1924547	5.19602792	105.7142857	100.7396422	110.7396422
20.5372781	0.19168126	5.21699402	107.1428571	101.5962604	111.5962604
20.7285782	0.19092112	5.23776534	108.5714286	102.4494588	112.4494588
20.9191246	0.19017386	5.25834622	110	103.2992959	113.2992959
21.1089301	0.18943912	5.27874084	111.4285714	104.1458283	114.1458283
21.2980069	0.18871652	5.29895324	112.8571429	104.9891109	114.9891109
21.4863671	0.18800571	5.31898733	114.2857143	105.8291971	115.8291971
21.6740222	0.18730636	5.33884688	115.7142857	106.6661389	116.6661389
21.8609835	0.18661815	5.35853554	117.1428571	107.4999865	117.4999865
22.0472621	0.18594076	5.37805684	118.5714286	108.3307889	118.3307889
22.2328685	0.1852739	5.39741418	120	109.1585937	119.1585937
22.4178133	0.18461729	5.41661088	121.4285714	109.9834473	119.9834473

Figure 5: Simulated D and Z functions for $P^d=5.38$ and $P^s=1.1/0.7*N^{0.3}$



Y(N)	MPL	Ps (=1.1/MPL)	Z (w=1.1)	D (=0.89*5.38*Y(N)+20)
19.9587228	0.19404314	5.668842541	113.142857	115.7811126
20.1523641	0.19324185	5.692348793	114.714286	116.7103894
20.3452113	0.1924547	5.715630711	116.285714	117.635855
20.5372781	0.19168126	5.738693423	117.857143	118.5575762
20.7285782	0.19092112	5.761541877	119.428571	119.4756177
20.9191246	0.19017386	5.784180842	121	120.3900424
21.1089301	0.18943912	5.806614924	122.571429	121.3009112
21.2980069	0.18871652	5.828848567	124.142857	122.2082833