Zinsabhängigkeit der Faustmann-Annuität

1. $$\frac{\partial Z_F}{\partial i} = \frac{\partial (i \cdot KW_F)}{\partial i} = \frac{\partial KW_F}{\partial i} \cdot i + KW_F$$

2. $$KW_F = \frac{G(t)}{e^{iL} - 1} = \frac{f(t) - Le^{it}}{e^{it} - 1}$$

3. $$\frac{\partial KW_F}{\partial i} = \frac{-tLe^{it}\left(e^{it} - 1\right) - te^{it}\left(f(t) - Le^{it}\right)}{\left(e^{it} - 1\right)^2} = \frac{te^{it}\left(L - f(t)\right)}{\left(e^{it} - 1\right)^2} =$$

$$= -\frac{\left(f(t) - L\right)te^{it}}{\left(e^{it} - 1\right)^2} < 0$$

4. $$\frac{\partial Z_F}{\partial i} = -\frac{\left(f(t) - L\right)te^{it}}{\left(e^{it} - 1\right)^2} \cdot i + \frac{f(t) - Le^{it}}{e^{it} - 1} =$$

$$= \frac{-f(t) + f(t)e^{it} + Le^{it} - Le^{it \cdot 2} + ite^{it}L - ite^{it}f(t)}{\left(e^{it} - 1\right)^2} =$$

$$= \frac{f(t)\left(e^{it} - 1\right) - f(t)ite^{it} - Le^{it}\left(e^{it} - 1\right) + Lite^{it}}{\left(e^{it} - 1\right)^2}$$

Zähler: $\underbrace{\left(f(t)-Le^{it}\right)}_{a}\underbrace{\left(e^{it}-1\right)}_{b}-\underbrace{ite^{it}}_{c}\underbrace{\left(f(t)-L\right)}_{d}$

5. $a \le d$

6. $b \overset{?}{\le} c$

- $i = 0:$ $\quad b = c$
- $i, t > 0:$ $\quad \frac{\partial b}{\partial i} = te^{it} < \frac{\partial c}{\partial i} = te^{it} + it^2 e^{it}$

$b \le c$

7. $ab - cd \le 0$

$\frac{\partial Z_F}{\partial i} \le 0$

8. $\lim_{i \to 0} Z_F = \lim_{i \to 0} i \cdot \frac{f(t)-Le^{it}}{e^{it}-1} = \frac{f(t)-L}{t}$